ESG and Mutual Fund Competition[⋆]

Ariadna Dumitrescu[a,∗], Javier Gil-Bazo[b]

[a] *ESADE Business School and University Ramon Llull, Av. Pedralbes, 60-62, 08034 Barcelona, Spain*
[b] *Universitat Pompeu Fabra and Barcelona Graduate School of Economics, c/Ramon Trias Fargas 25-27, 08005 Barcelona, Spain*

Abstract

We model competition among conventional funds and funds with diverse Environmental, Social, and Governance (ESG) mandates when investors have heterogeneous sustainability preferences. We characterize an equilibrium with segmentation, in which ESG funds cater exclusively to sustainability-minded investors, while conventional funds target only ESG-neutral investors. The model predicts that conventional funds survive only if they are of high quality. In contrast, underperforming ESG funds can persist due to the presence of investors with heterogeneous ESG preferences. Moreover, we show that competition from conventional funds is essential to curb the potential for ESG funds to set high fees and reduce investors' welfare. Our findings have implications for understanding ESG investor behavior, ESG fund performance, and the ongoing debate on the consequences of asset managers' sustainability mandates.

JEL codes: G11; G20; G23.

Keywords: ESG investors' preferences; ESG mutual funds; fund fees

[⋆] We are grateful for comments from Fernando Zapatero and participants at the 2024 Finance Theory Group Summer Conference and 31st Finance Forum. Javier Gil-Bazo acknowledges financial support from the Spanish Government, Ministry of Science and Innovation's grant PID2020-118541GB-I00, and Spanish Agencia Estatal de Investigación (AEI), through the Severo Ochoa Programme for Centres of Excellence in R&D (Barcelona School of Economics CEX2019-000915-S). Ariadna Dumitrescu acknowledges financial support from the Spanish Government, Ministry of Science and Innovation's grant PID2021-123748NB-I00, and Banc Sabadell.

[∗] Corresponding author

Email addresses: `ariadna.dumitrescu@esade.edu` (Ariadna Dumitrescu), `javier.gil-bazo@upf.edu` (Javier Gil-Bazo)

1. Introduction

Socially responsible investing (SRI) emerged in the 1970s as an investing strategy in which investors seek financial returns but screen out companies using ethical and moral criteria (such as tobacco, alcohol, weapons, etc.). ESG (Environmental, Social and Governance) investing builds on SRI and incorporates ESG criteria into the decision-making process. The asset management industry's embrace of ESG investment principles was initially relatively slow but has accelerated in recent years due to increasing firm and investor concerns about the impact of corporate activities on stakeholders and society more broadly. This growing interest has made mutual funds with ESG criteria one of the hottest investment trends with assets under management in funds using ESG considerations almost doubling from $22.8 trillion in 2016 to $35 trillion in 2020.[1] This trend has led to heightened competition among funds in the ESG space, but also between ESG funds and conventional funds. In this paper, we investigate how the coexistence of ESG funds with conventional funds affect the nature of mutual fund competition. In particular, we ask: How do ESG funds compete for investors with heterogenous ESG preferences? How do competition in the ESG space and in the conventional space affect each other? And, what is the impact on investors' welfare?

To achieve our goal, we build a model in which investors have heterogeneous preferences for sustainability and mutual funds compete to attract investors' money. In our model there are two types of funds: ESG funds, which consider ESG and financial criteria; and conventional funds, which consider only financial criteria. Both ESG and conventional funds are actively managed and can be of either high or low quality in terms of the asset manager's ability to deliver risk-adjusted returns (alpha). Funds with ESG mandates cater to investors with sustainability preferences by offering a range of approaches that align with investors' environmental, social, and governance preferences. In our model, investors' preferences for sustainability are heterogeneous in two different ways. First, we assume that while some investors, which we refer to as neutral investors, derive utility exclusively from alpha, other

[1] See Bloomberg Intelligence report, "ESG May Surpass $41 Trillion Assets in 2022, But Not Without Challenges," January 24, 2022, available at https://www.bloomberg.com/company/press/esg-may-surpass-41-trillion-assets-in-2022-but-not-without-challenges-finds-bloomberg-intelligence/.

investors, which we refer to as ESG investors, derive both pecuniary utility from alpha and non-pecuniary utility from investing according to sustainability principles. We assume that ESG investors' preference for sustainability is so strong that they shun conventional funds altogether.[2] Second, ESG investors also differ from each other in that they value different ESG objectives differently. For instance, some ESG investors may be primarily motivated by environmental concerns, while others may place more importance on social issues. Consequently, ESG investors derive more non-pecuniary utility from ESG funds whose mandates includes objectives that are more closely aligned with their own personal preferences. We further assume that investors can invest in risky assets only through mutual funds as in Gennaioli et al. (2015), so their utility of not investing is zero.

Our model delivers several predictions. First, in equilibrium low-quality conventional funds are driven out of the market by high-quality conventional funds. Second, there is segmentation in that ESG funds cater to ESG investors and refuse to compete for neutral investors, who invest only in high-quality conventional funds. Third, we derive the optimal fees and assets under management for all types of funds. We show that if the differentiation among ESG funds is important enough for ESG investors relative to performance differences, ESG funds of different quality will coexist in the ESG segment of the market and the low-quality funds will do so despite offering lower after-fee expected alpha. Therefore, the existence of investors' ESG preferences can explain the survival of ESG funds that are expected to underperform. This prediction is consistent with the empirical evidence that investors in ESG funds are less sensitive to fund underperformance (Bollen, 2007; Renneboog et al., 2011). Fourth, our model delivers a prediction related to cross sectional differences in performance. We should expect differences in performance among ESG funds but not among conventional funds. This is a natural consequence of segmentation. Competition is fierce in the conventional segment of the market, but is relaxed by investors' heterogeneous preferences in the ESG segment. To the extent that managerial skill persists, the model predicts that observed differences in performance among ESG funds also persist through time. In contrast, any

[2] ESG investors' disutility from investing in conventional funds is assumed to be infinite.

performance differences among conventional funds are the consequence of luck and short-lived.

Finally, the model yields a new prediction regarding strategic fee setting: ESG funds charge higher fees (and obtain higher mark-ups) on average than conventional funds. This is consistent with empirical evidence that investors in sustainable funds pay a "greenium" i.e., a higher fee, relative to investors in otherwise similar conventional funds (see Raghunandan and Rajgopal, 2022; Baker et al. 2022, and Huij et al., 2023).[3]

Our results carry potentially important implications. ESG mandates have become increasingly popular due to investors' growing interest in investments that reflect their values, but also to several initiatives aimed at strengthening asset managers' ESG commitment, such as the United Nations Principles for Responsible Investment Pledge and the Net Zero Asset Managers Initiative. The asset management industry has embraced with enthusiasm this commitment, although some large players have expressed disagreement in recent years.[4] Yet, a comprehensive evaluation of all the associated costs and benefits of ESG mandates is still pending. We contribute to this debate by showing that in an asset management market in which investors can invest only in ESG funds, fees would be higher than in a market where the two types of funds were available to investors. This results in diminished welfare for both neutral and ESG investors. The presence of conventional funds strengthens competition in the ESG segment of the market and thus, enhances the welfare of all investors.

Our model contributes to a line of theoretical research on mutual funds that departs from the perfectly competitive setting of Berk and Green (2004). These models study the impact of heterogeneous consumer tastes (Metrick and Zeckhauser, 1998, Massa; 2003; Hortaçsu and Syverson, 2004; Wahal and Wang, 2011; Khorana and Servaes, 2012; Hoberg et al., 2018; Kostovetsky and Warner, 2020), asymmetric information and different sensitivities to fees (Gil-Bazo and Ruiz-Verdú, 2008), heterogeneous liquidity needs (Nanda et al., 2000) and frictions (Dumitrescu and Gil-Bazo, 2018; Gârleanu and Pedersen, 2018; Roussanov et al.,

[3]See also Morningstar, 2020 U.S. Fund Fee Study that shows that the ESG funds' higher asset-weighted average expense ratio was 0.61% at the end of 2020 versus 0.41% for their conventional peers.

[4]See Financial Times, "Vanguard quits climate alliance in blow to net zero project," December 7, 2022, available at https://www.ft.com/content/48c1793c-3e31-4ab4-ab02-fd5e94b64f6b.

2021). Like those studies, ours explains why strategically-set fees may not offset differences in quality. The novelty of our model is that we introduce a new dimension along which investors differ: preferences for sustainable investing. Our paper shows that mutual funds that develop strategies in order to cater for heterogeneous preferences in terms of sustainability create product differentiation and this has important implications for fee setting, gross returns and asset allocation.

Our paper is related to the theoretical literature concerned with the impact of sustainability preferences on financial investment and asset prices.[5] Heinkel et al. (2001) show that polluting firms are pushed to reform because exclusionary screening negatively impacts their valuations. Pedersen et al. (2021) model ESG preferences and characterize an ESG-efficient frontier when the market is populated by ESG-motivated, ESG-aware and ESG-unaware investors. Pástor et al. (2021) show that the presence of investors who derive non-pecuniary utility from sustainable investing increases the prices of green stocks and decreases those of brown assets relative of standard model with no preferences for sustainability . In a model with partial segmentation, Zerbib (2022) shows that asset expected returns are affected by the tastes and exclusion strategies of heterogenous investors. Goldstein et al. (2021) analyse how investors' ESG preferences and the use of differential information alter the process of information aggregation in prices. We contribute to this literature by studying theoretically the consequences of investors' ESG preferences on mutual funds' expected returns.

Our paper is also related to the theoretical literature that investigates the influence of ESG investing on corporate behavior. Chowdhry et al. (2019) studies the optimal financing in the presence of externalities, highlighting the impact arising from the coexistence of profit and socially motivated investors on firms' profitability. The study underscores the necessity for impact investors to hold financial claims in order to incentivize profit-motivated investors to pursue social goals. Similarly, Oehmke and Opp (2022) study responsible investing in the presence of moral hazard, emphasizing the intricate interplay between production externalities and corporate financing constraints. Their findings indicate that responsible in-

[5] A large empirical literature has tested the predictions of the theory. See, for instance, Hong and Kacperczyk (2009), Luo and Balvers (2017), Zerbib (2022), Pástor et al. (2022).

vestors, by internalizing social externalities, play a pivotal role in fostering the expansion of the scaling of environmentally sustainable projects. Landier and Lovo (2020) build a general equilibrium model where sustainable investors yield equivalent returns to regular investors in a market subject to a search friction. Their analysis focuses on the strategy of an ESG fund that maximizes social welfare and shows that the presence of an ESG fund forces companies to partially internalize externalities and improve social welfare despite the selfishness of all agents. Finally, Green and Roth (2020) show that value-aligned investors who own socially valuable firms, compete against each other and push the price of firms upwards. We add to this literature by showing how investors' preferences for sustainability influence the strategic decisions of mutual funds.

The rest of the paper is organized as follows. In section 2, we present the theoretical framework of our analysis. In section 3, we present the equilibrium in three different markets with and without ESG and conventional funds. Section 4 compares these markets and discusses how the coexitence of ESG and conventional funds impacts competition and investor welfare. Finally, section 5 concludes. All proofs can be found in the Appendix.

2. The Model

Actively managed funds compete for investors' money. Funds can invest considering ESG issues or they can ignore them. We assume that actively managed funds may be of either high quality or low quality, reflecting manager's skill to earn positive risk-adjusted returns, or alpha. A fund's type is common knowledge. We denote the expected risk-adjusted return of high-quality and low-quality funds by R_H and R_L, respectively, with $R_H > R_L$. For simplicity, we refer to expected risk-adjusted return as return. We assume that there are two conventional funds, denoted by HC and LC, and those funds are of high and low quality, respectively, i.e. $R_{HC} = R_H$ and $R_{LC} = R_L$.

We consider two continuums of investors: investors with ESG preferences, with mass λ_P, and neutral investors, without ESG preferences, with mass λ_N. ESG investors prefer ESG funds over conventional funds, and among ESG funds, they prefer funds with an investment objective that is more aligned with their personal preferences. To model ESG investors' preference for a particular ESG fund, we build on Hotelling's (1929) model of horizontal differentiation and assume that ESG investors are uniformly distributed along a line of length λ_P with the two ESG funds located at the extremes of the line. Proximity of a fund to an investor measures the alignment between the fund's ESG objective and the investor's preference. The shorter the distance, the closer the alignment, and the higher the non-pecuniary utility derived by the investor from investing in that fund. We further assume that one of the two ESG funds, denoted by HS, is of higher quality than the other fund, denoted by LS (i.e., $R_{HS} = R_H$ and $R_{LS} = R_L$). The HS fund is located at $x = 0$, while the LS fund is located at $x = \lambda_P$.

We assume that an ESG investor's non-pecuniary utility of investing in an ESG fund that is a linear function of the investor's proximity to the fund. That is, an investor located at x derives non-pecuniary utility equal to $u_0 - kx$ from investing in the HS fund and non-pecuniary utility of $u_0 - k(\lambda_P - x)$ from investing in the LS fund, with $u_0 > 0$. Notice that when the investor's preferences are perfectly aligned with the fund's objective, the misalignment cost is 0. This is the case for the investor located at $x = 0$ investing in fund HS and for the investor located at $x = \lambda_P$ investing in fund LS. The parameter k can be interpreted as the intensity of the investor's ESG preference for a specific ESG dimension. When $k = 0$, ESG investors derive the same non-pecuniary utility from investing in either fund.[6] We assume that the misalignment between ESG investors' preferences and conventional funds is so large that ESG investors suffer a high disutility from investing in those funds and avoid them altogether. This implies that ESG investors are restricted to investing in one of the two ESG funds. Neutral investors may invest in any fund and their utility depends only on the net return of the fund they invest in. Both neutral and ESG investors have a reservation utility of zero, corresponding to the utility of not investing in any fund.[7]

[6] Similarly to the standard interpretation of the Hotelling model, the "distance" between the two funds can represent the difference in investment objectives between the two funds, not geographical distance.

[7] As in the model of Gennaioli et al. (2015), we assume that investors cannot directly invest in stocks, which can be explained if investors cannot take risks without the advice of asset managers.

2.1. The investor's problem

Each investor is endowed with one dollar and chooses one fund. The fund charges a fee, f, per dollar invested. Neutral investors derive a utility equal to $U^N_\varphi = R_\varphi - f_\varphi$, for investing in the active mutual fund of type $\varphi \in \{HS, LS, HC, LC\}$, where HS and LS denote ESG funds with different ESG integration strategy, HC a high quality conventional fund and LC a low quality conventional fund. The ESG investor i is willing to invest only in ESG funds and derives a utility equal to $U^{ESG}_{i,\varphi} = R_\varphi - f_\varphi + (u_0 - kd_{i,\varphi})$, where $d_{i,\varphi}$ denotes the distance between the investor and the fund. An investor decides to invest with fund φ as long as the utility from investing with that fund is higher than the utility from investing with any other fund and higher than her reservation utility.

The demand of neutral investors is split equally across all active funds offering the highest positive net-of-fee return and is zero for all the other funds.

The demand of ESG investors for conventional funds is zero by assumption. Each ESG investor chooses to invest in the ESG fund offering the highest utility given the fund's return, fee, and the proximity between the fund and the investor, as long as this utility is positive, otherwise she decides not to invest. In case that both funds offer the same positive utility, her wealth is split equally between them.

2.2. The fund manager's problem

Fund managers choose the fees that maximize their profits given investors' demand functions and the other managers' strategies. Without loss of generality, we assume that the marginal cost to the manager of operating the fund is zero. Therefore, the manager's problem becomes:

$$\max_{f_\varphi} \Pi_\varphi = f_\varphi \left(q_{P,\varphi} + q_{N,\varphi} \right),$$

where $q_{P,\varphi}$, $q_{N,\varphi}$ denote the total demand for the fund from ESG and neutral investors, respectively in fund $\varphi \in \{HS, LS, HC, LC\}$.

3. Equilibrium

3.1. Market with only conventional funds

We consider first the case when only conventional funds exists. Thus, there exists a high quality conventional fund that has a return R_H and a low quality conventional fund that has a return R_L. The ESG investors are not willing to invest in any conventional fund and their utility is therefore equal to 0. As a result, the conventional funds compete only for neutral investors. Since the fund's type and return are common knowledge, the two funds compete à la Bertrand. Since the returns are such that $R_H > R_L$, the HC fund can drive the LC fund out of the market simply by setting a fee such that $R_H - f_{HC} > R_L$. In that case, LC cannot set a positive fee and stay in business. Therefore, the HC fund gains all the market of neutral investors and the LC fund does not operate. The high quality fund will choose $f_{HC} = \Delta$, where $\Delta \equiv R_H - R_L$. All neutral investors invest with the high-quality conventional fund and their utility is equal to $U^N_{HC} = R_H - f_{HC} = R_H - \Delta = R_L$.

3.2. Market with both ESG and conventional funds

The two ESG funds, HS and LS, are horizontally differentiated products for ESG investors because of their different ESG strategy. The two conventional funds, HC and LC, on the other hand, are not differentiated in that they are perfect substitutes for neutral investors. Just as in the previous case, the conventional funds engage in Bertrand-like competition. The HC fund sets a fee $f_{HC} < \Delta$, and the LC is driven out of the market.

Lemma 1. *In equilibrium, no investors choose to invest in the low quality conventional (LC) fund.*

In next lemma, we show that neutral investors do not invest in ESG funds, either.

Lemma 2. *In equilibrium, ESG funds cater to ESG investors and conventional funds cater to neutral investors.*

To see how this form of segmentation arises in equilibrium, assume that ESG funds decide to compete against conventional funds for neutral investors. For the same reasons that LC is driven out of the market, LS does not attract any neutral investors. If the two high-quality funds, HS and HC, engage in price competition, since both funds offer the same return R_H, each has an incentive to undercut its fee until the fee equals zero, i.e., $f_{HC} = f_{HS} = 0$. At that fee, both the HC and HS funds make zero profits. However, the HS fund can always charge an arbitrarily small but positive fee and make positive profits by operating only with ESG investors. More specifically, it is sufficient for the HS fund to charge a fee lower than $\Delta \equiv R_H - R_L$ to dominate the LS fund for investors who have a sufficiently high preference for HS regardless of the fee charged by the LS fund. This ensures that those investors that are closest to HS are willing to invest in it, and therefore HS makes a positive profit. Consequently, in equilibrium HS does not sell to neutral investors.

The two ESG funds, HS and LS, compete against each other only for ESG investors. When all ESG investors purchase ESG funds, the market is covered. When some ESG investors decide not to invest, the market is not covered and the two ESG funds act as local monopolies. Depending on the values of the model parameters, we have three possible situations. First, the market is covered and all ESG investors invest in the HS fund. This happens when k is small enough, i.e., when investors exhibit less intense preferences for a particular ESG strategy so the two ESG funds are closer substitutes. Second, the market is covered and the two ESG funds share the market of ESG investors, each catering to the set of investors with more aligned preferences. This happens for an intermediate value of k. Third, when k is very high, the market is not covered. Some investors leave the market, funds act as local monopolies and set fees accordingly. Consequently, heterogeneity in investors' preferences for ESG allow both HS and LS funds to coexist in equilibrium. It is also necessary that the difference in performance is not too large, as large differences in performance lead all investors to concentrate in the best performing fund. These results are presented in the following Proposition, where for convention we set the fees of funds that stay out of the market equal to 0:

Proposition 1. *In equilibrium, the fees charged by the funds are the following*

$$f^*_{HS} = \begin{cases} \Delta - k\lambda_P & if \quad k \leq k_1, \\ k\lambda_P + \frac{1}{3}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2}(R_H + u_0) & if \quad k \geq k_2, \end{cases}$$

$$f^*_{LS} = \begin{cases} 0 & if \quad k \leq k_1, \\ k\lambda_P - \frac{1}{3}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2}(R_L + u_0) & if \quad k \geq k_2, \end{cases}$$

$$f^*_{HC} = \begin{cases} \frac{\lambda_P}{\lambda_N + \lambda_P}(\Delta - k\lambda_P) - \varepsilon & if \quad k \leq k_1, \\ \min\{\Delta, f^*\} - \varepsilon & if \quad k_1 < k < k_2, \\ \min\{\Delta, f^{**}\} - \varepsilon & if \quad k \geq k_2, \end{cases}$$

$$f^*_{LC} = 0,$$

and the quantities invested in each fund are

$$q^*_{N,HC} = \lambda_N, \; q^*_{N,LC} = 0,$$

$$q^*_{P,HC} = q^*_{P,LC} = 0,$$

$$q^*_{P,HS} = q^*_{HS} = \begin{cases} \lambda_P & if \quad k \leq k_1, \\ \frac{1}{2k}\left(k\lambda_P + \frac{1}{3}\Delta\right) & if \quad k_1 < k < k_2, \\ \frac{1}{2k}(R_H + u_0) & if \quad k \geq k_2, \end{cases}$$

$$q^*_{P,LS} = q^*_{LS} = \begin{cases} 0 & if \quad k \leq k_1, \\ \frac{1}{2k}\left(k\lambda_P - \frac{1}{3}\Delta\right) & if \quad k_1 < k < k_2, \\ \frac{1}{2k}(R_L + u_0) & if \quad k \geq k_2, \end{cases}$$

where ε is strictly positive and arbitrarily small, $k_1 \equiv \frac{\Delta}{3\lambda_P}$, $k_2 \equiv \frac{2u_0 + R_H + R_L}{3\lambda_P}$ and the definitions of f^* and f^{**} are provided in the Appendix.

In the cases in which the LS fund survives, i.e., when $k > k_1$, it charges a lower fee than the HS fund. Moreover, when $k_1 < k < k_2$, the difference in fees between both funds equals

$\frac{2}{3}\Delta = \frac{2}{3}(R_H - R_L)$. When $k \geq k_2$, the difference is $\frac{1}{2}(R_H - R_L)$. In either case, the difference in fees is not enough to fully offset the difference in before-fee performance. Therefore, when the LS fund survives in the market, it does so despite offering lower net performance than the high quality ESG fund, HS. In other words, in equilibrium, we expect differences in after-fee returns between ESG funds. In contrast, conventional funds are not expected to exhibit any differences in performance.

Notice that our model is static. However, since there is no asymmetric information and therefore, no role for learning about performance, a dynamic model built as a succession of static models would yield an identical outcome provided that pre-fee performance does not change over time. This observation helps understand why differences in net performance can persist. Performance persistence in our model is the consequence of variety of ESG styles and preferences in the ESG segment of the market. If funds were identical in the ESG dimension or if ESG investors all valued the same ESG attributes ($k = 0$), competition between ESG funds would drive the lower quality one out of the market, just like in the conventional segment of the market.

Figure 1 shows the equilibrium fees of the high-quality ESG fund and the high-quality ESG fund as a function of k. In order to ensure that the HS fund does not want to deviate from the equilibrium with segmentation, the HC fund always sets a fee lower than that charged by HS so that the profit obtained by HS from serving only ESG investors is higher than the profit obtained by lowering the fee in order to serve also the entire market of neutral investors. Therefore, although the markets are segmented, the fees in the conventional segment are influenced by competition from ESG funds.

When the ESG market is covered, $k_1 < k < k_2$, the average fee in the ESG segment is higher than or equal to the fee in the conventional segment of the market: $\frac{f_{HS} + f_{LS}}{2} \geq f_{HC}$. This result provides an equilibrium explanation to the findings of Raghunandan and Rajgopal (2022), Baker et al. (2022) and Huij et al. (2023) that ESG funds charge higher management fees than conventional funds. More specifically, Raghunandan and Rajgopal (2022) show that non-low carbon funds underperform financially and charge higher management fees relative to the non-ESG funds (about 1.5 basis points higher), and Baker et al. (2022) show that ESG funds charge on average 5.9 basis points higher fees. A similar result is obtained by Huij et al. (2023) who show that green funds charge about 5 basis points higher fees (and this difference can go up to 10 basis points for a low -carbon portfolio).

When the strength of ESG investors' preferences is weak enough (i.e., $k \leq k_1$), k is small relative to the difference in returns between the LS and HS funds, and fees decrease with k. In this case, in order to keep the LS fund out of the market, the HS fund is forced to lower the fee as k increases. Otherwise, some investors will switch to LS.

When k is high enough, $k_1 < k < k_2$, the market is covered by the two funds. In this case, the horizontal differentiation between the two funds given investors' preferences, is higher than the cost of investing with a less-than perfectly aligned fund. As the value of k rises, the two funds engage in less intense competition for the same investors, i.e., the market power effect dominates the effect of competition between two funds. Thus, investors closer to a given fund become captive giving the fund monopoly power, which in turn allows the fund to increase the fee. As k increases even more, $k > k_2$, some investors, too misaligned with both ESG funds, decide to leave the market. The demand for each ESG fund decreases and the only effect left is the market power effect.

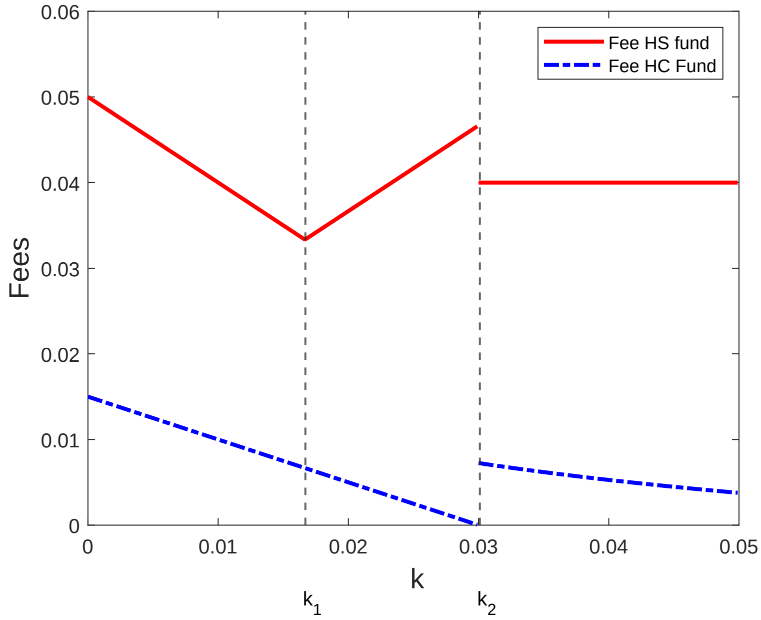

Figure 1: Fees of the HS fund and the HC fund. Parameter values: $R_H = 6\%$, $R_L = 1\%$, $\lambda_P = 1$, $\lambda_N = 1$ and $u_0 = 0.02$.

3.3. Market with only ESG funds

We now study a market in which only ESG funds are offered to all types of investors. As mentioned above, the two ESG funds, HS and LS, are differentiated products from the point of view of ESG investors because of their particular ESG objectives. Let us first consider the case when there are no neutral investors. In this case an equilibrium always exists and it is characterized in the following lemma.

Lemma 3. *When there are only ESG investors in the market, the equilibrium fees are*

$$f^*_{0HS} = \begin{cases} \Delta - k\lambda_P & if \quad k \leq k_1, \\ k\lambda_P + \frac{1}{3}\Delta & if \quad k_1 < k \leq k_2, \\ \frac{1}{2}(R_H + u_0) & if \quad k > k_2, \end{cases}$$

$$f^*_{0LS} = \begin{cases} 0 & if \quad k \leq k_1, \\ k\lambda_P - \frac{1}{3}\Delta & if \quad k_1 < k \leq k_2, \\ \frac{1}{2}(R_L + u_0) & if \quad k > k_2, \end{cases}$$

and the equilibrium quantities are

$$q^*_{0HS} = \begin{cases} \lambda_P & if \quad k \leq k_1, \\ \frac{1}{2}\lambda_P + \frac{1}{6k}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2k}(R_H + u_0) & if \quad k \geq k_2, \end{cases}$$

$$q^*_{0LS} = \begin{cases} 0 & if \quad k \leq k_1, \\ \frac{1}{2}\lambda_P - \frac{1}{6k}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2k}(R_L + u_0) & if \quad k \geq k_2. \end{cases}$$

All ESG investors choose to invest only with the high quality fund when the intensity of ESG preferences, k, is low. Similarly to the case when there are only conventional funds, the HS fund is able to drive out of the market the LS fund. When k takes an intermediate value, the two funds share the market. Finally, when k is high, the market is not covered and both funds act as local monopolies. The equilibrium fees and quantities for the ESG funds in this case are exactly the same as the ones when both ESG and conventional funds exists. In that case, due to segmentation of investors, ESG funds were competing only for attracting the ESG investors.

The existence of a positive mass of neutral investors changes the equilibrium in the market for ESG funds in a fundamental way, as funds also compete this mass of investors. Since no conventional funds are available, the neutral investors choose among the ESG funds the one that offers them the highest possible net return.

Proposition 2. *If the mass of neutral investors is high* $\left(\lambda_N \geq \frac{\Delta}{k}\right)$ *there is no equilibrium. When the mass of neutral investors is low* $\left(\lambda_N < \frac{\Delta}{k}\right)$ *the fees charged by the funds are the following:*

$$f_{HS}^{O*} = \begin{cases} \Delta - k\lambda_P & \text{if} \quad k \leq k_1^O, \\ k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta & \text{if} \quad k_1^O < \ k \leq \min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\}, \\ \frac{1}{2}\left(R_H + u_0 + k\lambda_N\right) & \text{if} \quad k_2^O < \ k \leq \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}, \end{cases}$$

$$f_{LS}^{O*} = \begin{cases} 0 & \text{if} \quad k \leq k_1^O, \\ k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta & \text{if} \quad k_1^O < \ k \leq \min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\} \\ \frac{1}{2}\left(R_L + u_0\right) & \text{if} \quad \begin{array}{c} k > k_2^O \text{ and } k < \frac{\Delta}{\lambda_N} \\ k_2^O < \ k \leq \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\} \end{array} \end{cases}$$

and the equilibrium quantities

$$q_{HS}^{O*} = \begin{cases} \lambda_P + \lambda_N & \text{if} \quad k \leq k_1^O, \\ \frac{1}{2}\lambda_P + \frac{2}{3}\lambda_N + \frac{1}{6k}\Delta & \text{if} \quad k_1^O < \ k \leq \min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\}, \\ \frac{1}{2k}\left(R_H + u_0 + k\lambda_N\right) & \text{if} \quad k_2^O < \ k \leq \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}, \end{cases}$$

$$q_{LS}^{O*} = \begin{cases} 0 & \text{if} \quad k \leq k_1^O, \\ \frac{1}{2}\lambda_P + \frac{1}{3}\lambda_N - \frac{1}{6k}\Delta & \text{if} \quad k_1^O < \ k \leq \min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\}, \\ \frac{1}{2k}\left(R_L + u_0\right) & \text{if} \quad k_2^O < \ k \leq \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}, \end{cases}$$

where $k_1^O \equiv \frac{\Delta}{2\lambda_N + 3\lambda_P}$, $k_2^O \equiv \frac{2u_0 + R_H + R_L}{2\lambda_P + \lambda_N}$. *No equilibrium exists if* $\min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\} < k \leq k_2^O$ *and* $k > \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}$.

When the mass of neutral investors is large, there is no equilibrium. Both funds have incentives to undercut each other's fee and get the whole market of neutral investors. In the absence of ESG investors, this would drive the LS fund out of the market. However, LS can always charge a positive fee to ESG investors who are the most aligned with its ESG objective.

But if it did, then the HS would raise its fee, which would give incentives to LS to deviate, lower its fee, and gain the whole neutral segment of the market. Therefore, when there are many neutral investors, there is no pair of fees, f_{HS}, f_{LS}, that is incentive compatible.

The fact that $k_1^O \leq k_1$ and $k_2^O \leq k_2$ shows that competition in the case only ESG funds exist is relaxed and HS fund can charge higher fees as more investors are willing to invest with it.

4. Discussion

The asset management industry currently faces an important transformation as environmental and social issues gain prominence, leading to the integration of ESG factors into the funds' investment processes and strategies. In response to the increasing demand for ESG investments, asset managers have launched new funds. According to Morningstar, the number of funds with sustainable mandates that were launched in the U.S. hit a record level (119 funds in 2021, 103 funds in 2022 and 67 funds in 2023).[8] Globally, ESG assets are expected to exceed \$53 trillion by 2025, representing more than a third of the \$140.5 trillion in projected total assets under management.[9] Given this context, it seems pertinent to consider the potential consequences of a market structure where only funds with an ESG mandate exist. How would this impact mutual fund competition and investor welfare?

In order to shed light on this question, we use our model to contrast the equilibrium fees and investors' welfare between a scenario featuring both conventional funds and ESG funds and one where only ESG funds are available. Notice first that the absence of conventional funds can lead to situations when the equilibrium fails to exist. As explained above, if there are only ESG funds in the market and the mass of neutral investors is high relative to differences in performance of the two funds (for example), the market break downs. The existence of conventional funds in the market in this situation solves this problem and, therefore, improves the welfare of all market participants.

[8] "Global Sustainable Fund Flows: Q1 2024 in Review," report available at https://www.morningstar.com/lp/global-esg-flows.

[9] See https://www.bloomberg.com/professional/blog/esg-assets-may-hit-53-trillion-by-2025-a-third-of-global-aum/

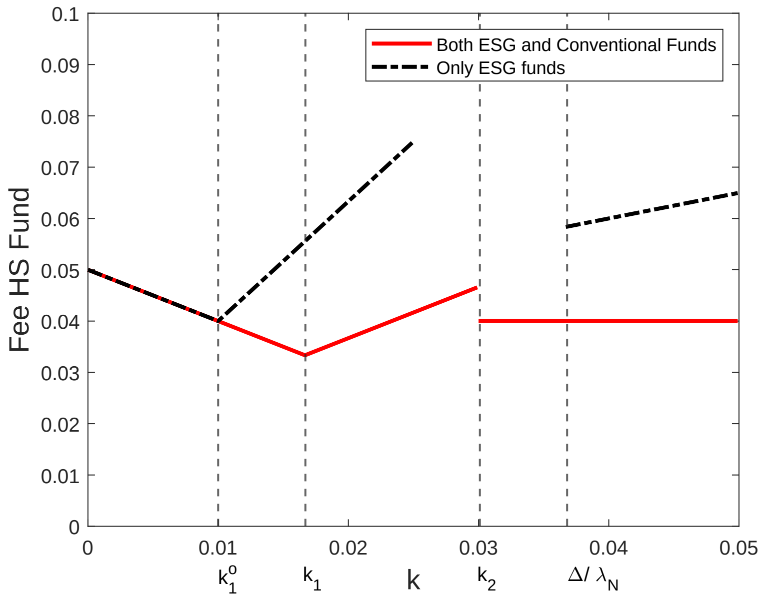

Figure 2: Fees of the HS fund in an economy where both ESG and conventional funds exist and in an economy where only ESG funds exist. Parameter values: $R_H = 6\%$, $R_L = 1\%$, $\lambda_P = 1$, $\lambda_N = 1$ and $u_0 = 0.02$.

Let us next consider the equilibrium with only ESG funds catering to ESG and neutral investors and compare it to the equilibrium where both ESG and conventional funds compete.

Proposition 3. *The fee of the high quality ESG fund (HS fund) in an economy where only ESG funds exists is higher than in the case where conventional and ESG funds coexist.*

We have shown that $f^O_{HS} \geq f_{HS}$ and that $f_{HS} > f_{HC}$. As a result, the fee charged to neutral investors where there are only ESG funds is higher than or equal to the fee when both ESG and conventional funds are available (see Figure 2).

Proposition 4. *Investors' welfare in an economy where only ESG funds exists is lower than in the case when both conventional and ESG funds coexist.*

Notice that the welfare of the neutral investors is always strictly lower in the case where only ESG funds exists (see Figure 3). When the misalignment cost is low ($k < k_1$) the fees and

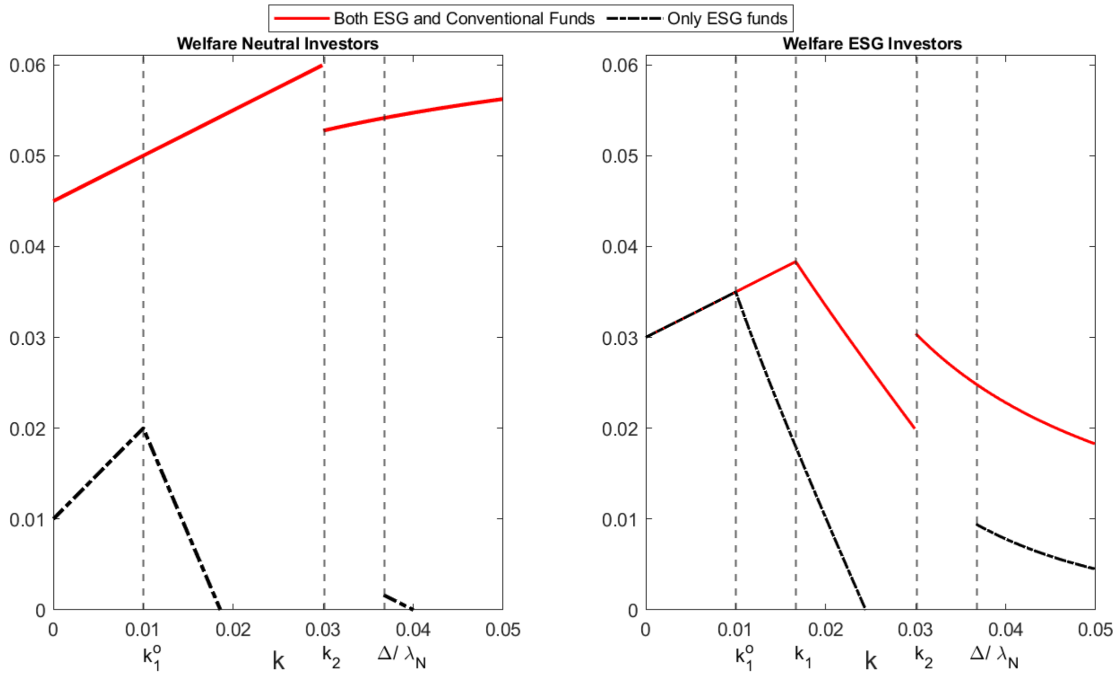

Figure 3: Investors' welfare in an economy where both ESG and conventional funds exist and in an economy where only ESG funds exist. Parameter values: $R_H = 6\%$, $R_L = 1\%$, $\lambda_P = 1$, $\lambda_N = 1$ and $u_0 = 0.02$.

the quantities are equal in both cases and therefore the welfare of ESG investors is exactly the same. In all the other cases, the welfare of ESG investors is higher when ESG and conventional funds coexist in the market.

Consequently, competition from conventional funds not only improves the welfare of investors without a preference for sustainability, but it also helps ESG investors, as it reduces the equilibrium fees of ESG funds.

5. Conclusions

In this paper, we study how competition among mutual funds is shaped by the presence of investors with sustainability preferences and mutual funds that cater to those preferences. To achieve our goal, we construct a model of the mutual fund market where active funds with different ESG objectives and different quality compete for investors' money. Investors have heterogenous preferences for ESG. Not all investors care for sustainability, and among those who do, they value different ESG objectives differently. The model predicts that in equilibrium the market is segmented: neutral investors (those with no preference for ESG) invest only in conventional funds and ESG investors invest only in ESG funds. While competition is fierce in the conventional segment of the market and only the best funds survive, it is relaxed by investors' ESG preferences in the ESG segment of the market. If the intensity of ESG investors' preferences is sufficiently high, ESG funds of lower quality will be able to survive.

Since there is no asymmetric information in our model, there is no role for learning either, so the outcome of a multi-period version of the model is identical to that of the static version presented in the paper. This implies that, to the extent that managerial skill persists, performance differences among ESG funds will also persist through time. Among conventional funds, however, competition ensures the homogeneity of operating funds, so one would not expect to observe differences in performance.

Our results unveil a form of segmentation that arises as a consequence of both investor heterogeneity in their ESG preferences and product differentiation along the ESG dimension. ESG acts as a friction that relaxes competition between mutual funds in terms of performance and fees. The consequences are important. First, investors may end up investing in products with lower financial returns and paying prices that do not offset quality differences. Second, forcing all asset managers in the economy to commit to ESG investing has a negative impact on the welfare of both conventional and ESG investors. Therefore, our analysis suggests that public and private initiatives aimed at making ESG considerations a common objective in delegated asset management has potentially important costs for investors that cannot be excluded from the ongoing debate.

References

Baker, M., M. L. Egan, and S. K. Sarkar (2022). How do investors value ESG? Technical report, National Bureau of Economic Research.

Berk, J. and R. Green (2004). Mutual fund flows and performance in rational markets. *Journal of Political Economy 112*(6), 1269–1295.

Bollen, N. P. (2007). Mutual fund attributes and investor behavior. *Journal of Financial and Quantitative Analysis 42*(3), 683–708.

Chowdhry, B., S. W. Davies, and B. Waters (2019). Investing for impact. *Review of Financial Studies 32*(3), 864–904.

Dumitrescu, A. and J. Gil-Bazo (2018). Market frictions, investor sophistication, and persistence in mutual fund performance. *Journal of Financial Markets 40*, 40–59.

Gârleanu, N. and L. H. Pedersen (2018). Efficiently inefficient markets for assets and asset management. *The Journal of Finance 73*(4), 1663–1712.

Gennaioli, N., A. Shleifer, and R. Vishny (2015). Money doctors. *Journal of Finance 70*(1), 91–114.

Gil-Bazo, J. and P. Ruiz-Verdú (2008). When cheaper is better: Fee determination in the market for equity mutual funds. *Journal of Economic Behavior & Organization 67*(3), 871–885.

Goldstein, I., A. Kopytov, L. Shen, and H. Xiang (2021). On ESG investing: Heterogeneous preferences, information, and asset prices. *Working Paper Available at SSRN: https://ssrn.com/abstract=3823042*.

Green, D. and B. Roth (2020). The allocation of socially responsible capital. *Working Paper Available at SSRN: https://ssrn.com/abstract=3737772*.

Heinkel, R., A. Kraus, and J. Zechner (2001). The effect of green investment on corporate behavior. *Journal of Financial and Quantitative Analysis 36*(4), 431–449.

Hoberg, G., N. Kumar, and N. Prabhala (2018). Mutual fund competition, managerial skill, and alpha persistence. *Review of Financial Studies 31*(5), 1896–1929.

Hong, H. and M. Kacperczyk (2009). The price of sin: The effects of social norms on markets. *Journal of Financial Economics 93*(1), 15–36.

Hortaçsu, A. and C. Syverson (2004). Search costs, product differentiation, and welfare effects of entry: A case study of SP 500 Index Funds. *Quarterly Journal of Economics 119*(4), 403–456.

Hotelling, A. (1929). Stability in competition. *Economic Journal 39*, 41–57.

Huij, J., D. Laurs, P. Stork, and R. C. Zwinkels (2023). Are Investors Paying to be Green? Evidence from Mutual Funds. *Working Paper*.

Khorana, A. and H. Servaes (2012). What drives market share in the mutual fund industry? *Review of*

Finance 16(1), 81–113.

Kostovetsky, L. and J. B. Warner (2020). Measuring innovation and product differentiation: Evidence from mutual funds. *Journal of Finance 75*(2), 779–823.

Landier, A. and S. Lovo (2020). ESG investing: How to optimize impact? *Working Paper Available at SSRN: https://ssrn.com/abstract=3508938*.

Luo, H. A. and R. J. Balvers (2017). Social screens and systematic investor boycott risk. *Journal of Financial and Quantitative Analysis 52*(1), 365–399.

Massa, M. (2003). How do family strategies affect fund performance? When performance-maximization is not the only game in town. *Journal of Financial Economics 67*(2), 249–304.

Metrick, A. and R. Zeckhauser (1998). Price versus quantity: Market-clearing mechanisms when consumers are uncertain about quality. *Journal of Risk and Uncertainty 17*(3), 215–243.

Nanda, V., M. Narayanan, and V. A. Warther (2000). Liquidity, investment ability, and mutual fund structure. *Journal of Financial Economics 57*(3), 417–443.

Oehmke, M. and M. M. Opp (2022). A theory of socially responsible investment. *Swedish House of Finance Research Paper* (20-2).

Pástor, L., R. F. Stambaugh, and L. A. Taylor (2021). Sustainable investing in equilibrium. *Journal of Financial Economics 142*(2), 550–571.

Pástor, L., R. F. Stambaugh, and L. A. Taylor (2022). Dissecting green returns. *Journal of Financial Economics 146*(2), 403–424.

Pedersen, L. H., S. Fitzgibbons, and L. Pomorski (2021). Responsible investing: The ESG-efficient frontier. *Journal of Financial Economics 142*(2), 572–597.

Raghunandan, A. and S. Rajgopal (2022). Do ESG funds make stakeholder-friendly investments? *Review of Accounting Studies 27*(3), 822–863.

Renneboog, L., J. Ter Horst, and C. Zhang (2011). Is ethical money financially smart? Nonfinancial attributes and money flows of socially responsible investment funds. *Journal of Financial Intermediation 20*(4), 562–588.

Roussanov, N., H. Ruan, and Y. Wei (2021). Marketing mutual funds. *The Review of Financial Studies 34*(6), 3045–3094.

Wahal, S. and A. Y. Wang (2011). Competition among mutual funds. *Journal of Financial Economics 99*(1), 40–59.

Zerbib, O. D. (2022). A sustainable capital asset pricing model (s-capm): Evidence from environmental integration and sin stock exclusion. *Review of Finance 26*(6), 1345–1388.

Appendix

Proof of Proposition 1. The manager of each fund φ, with $\varphi \in \{HS, LS, HC, GB\}$ solves the following maximization problem

$$\max_{f_\varphi \geq 0} \Pi_\varphi = f_\varphi \left(q_{P,\varphi} + q_{N,\varphi} \right).$$

We have shown that there is segmentation: neutral investors do not invest in conventional funds, and therefore $q_{P,\varphi} = 0$, for $\varphi \in \{HC, LC\}$.

Therefore, we solve for the optimal strategy of HS and LS funds under segmentation. The ESG investor i decides to invest in fund HS if her participation constraint is satisfied, i.e. $U_{i,HS}^{ESG} \equiv U_{HS} \geq 0$. We denote by x the demand of investors for fund HS. Since investors have each \$1 to invest, x is also the location of the investor that is indifferent between investing or not with the fund. Similarly, an investor j decides to invest in fund LS if her participation constraint is satisfied, i.e. $U_{j,LS}^{ESG} \equiv U_{LS} \geq 0$. We denote by y the demand of investors for fund LS, and the location of the last investor willing to invest in fund LS is $y = \lambda_P - x$. The manager of fund HS solves the following maximization problem

$$\begin{aligned} \max_{f_{HS}} \Pi_{HS} &= f_{HS} x \\ s.t.\ U_{HS} &= R_H - f_{HS} + (u_0 - kx) \geq 0 \\ x + y &\leq \lambda_P, \\ f_{HS} &\geq 0, \end{aligned}$$

and the manager of fund LS solves

$$\begin{aligned} \max_{f_{LS}} \Pi_{LS} &= f_{LS} y \\ s.t.\ U_{LS} &= R_L - f_{LS} + (u_0 - ky) \geq 0 \\ x + y &\leq \lambda_P, \\ f_{LS} &\geq 0. \end{aligned}$$

The Lagrangean for the problem of the manager of the HS fund is

$$\mathcal{L}_{HS} = f_{HS}x - \mu_{HS}\left(-\left(u_0 - kx\right) - R_H + f_{HS}\right) - \eta\left(x + y - \lambda_P\right),$$

and the one for the problem of the manager of the LC fund is

$$\mathcal{L}_{LS} = f_{LS}y - \mu_{LS}\left(-\left(u_0 - ky\right) - R_L + f_{LS}\right) - \eta\left(x + y - \lambda_P\right).$$

The Kuhn-Tucker conditions for these problems are

$$\begin{aligned}
\frac{\partial \mathcal{L}_\varphi}{\partial f_\varphi} &\leq 0, \quad f_\varphi \geq 0, \quad f_\varphi \frac{\partial \mathcal{L}_\varphi}{\partial f_\varphi} = 0, \\
U_\varphi &= R_\varphi - \ f_\varphi + (u_0 - kx) \geq 0, \quad \mu_\varphi \geq 0, \ \mu_\varphi\left(-\left(u_0 - kx\right) - R_\varphi + f_\varphi\right) = 0, \\
x + y &\leq \lambda_P, \ \eta \geq 0, \ \eta\left(x + y - \lambda_P\right) = 0\,,
\end{aligned} \tag{1}$$

for $\varphi \in \{HS, LS\}$.

Since the manager's profit in case $f_\varphi = 0$ equals 0, we consider only the cases when $f_\varphi > 0$. Therefore, we have

$$\begin{aligned}
\frac{\partial \mathcal{L}_{HS}}{\partial f_{HS}} &= x + f_{HS}\frac{\partial x}{\partial f_{HS}} - \mu_{HS}\left(k\frac{\partial x}{\partial f_{HS}} + 1\right) - \eta\frac{\partial x}{\partial f_{HS}} = 0, \\
\frac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} &= y + f_{LS}\frac{\partial y}{\partial f_{LS}} - \mu_{LS}\left(k\frac{\partial y}{\partial f_{LS}} + 1\right) - \eta\frac{\partial y}{\partial f_{LS}} = 0.
\end{aligned} \tag{2}$$

Case I $\mu_{HS} = \mu_{LS} = \eta = 0$

The Kuhn-Tucker conditions (1) imply that in this case

$$\begin{aligned}
x + f_{HS}\frac{\partial x}{\partial f_{HS}} &= 0, \\
y + f_{LS}\frac{\partial y}{\partial f_{LS}} &= 0, \\
U_{HS} &\geq 0, \ U_{LS} \geq 0 \text{ and } x + y \leq \lambda_P.
\end{aligned}$$

Case I.1 If $x + y < \lambda_P$ then $U_{HS} = U_{LS} = 0$ and from here combined with the conditions

(2) we obtain that both funds act as local monopolies and therefore

$$\begin{aligned} f_{HS} &= \frac{R_H + u_0}{2}, \\ f_{LS} &= \frac{R_L + u_0}{2}, \\ x &= \frac{R_H + u_0}{2k}, \\ y &= \frac{R_L + u_0}{2k}. \end{aligned}$$

Since $x + y < \lambda_P$, but this is possible if and only if $\frac{R_H + u_0}{2k} + \frac{R_L + u_0}{2k} < \lambda_P$ or equivalently $k > \frac{1}{2\lambda_P}\left(2u_0 + R_L + R_H\right)$.

Case I.2 If $x + y = \lambda_P$ the ESG investors decide whether to invest in the HS fund or the LS fund depending on their location. In this case the market is covered and the two funds compete for attracting the investors. Thus, we have that the marginal investor's location, x, satisfies

$$R_H - f_{HS} + (u_0 - kx) = R_L - f_{LS} + (u_0 - k(\lambda_P - x)).$$

Therefore the marginal investor is located at

$$x = \frac{\lambda_P}{2} + \frac{R_H - f_{HS} - (R_L - f_{LS})}{2k} = \frac{\lambda_P}{2} + \frac{r_{HS} - r_{LS}}{2k}, \tag{3}$$

where $r_{HS} \equiv R_H - f_{HS}$ and $r_{LS} \equiv R_L - f_{LS}$.

It follows

$$\begin{aligned} q_{HS} &= \frac{\lambda_P}{2} + \frac{r_{HS} - r_{LS}}{2k}, \\ q_{LS} &= \frac{\lambda_P}{2} - \frac{r_{HS} - r_{LS}}{2k}. \end{aligned}$$

Therefore the conditions (2) become

$$\begin{aligned} \frac{\lambda_P}{2} + \frac{r_{HS} - r_{LS}}{2k} + \left(-\frac{1}{2k}\right) f_{HS} &= 0, \\ \frac{\lambda_P}{2} - \frac{r_{HS} - r_{LS}}{2k} + \left(-\frac{1}{2k}\right) f_{LS} &= 0. \end{aligned}$$

Solving for f_{HS} and f_{LS} and imposing that fees are positive we find that in equilibrium

$$
\begin{aligned}
f_{HS} &= \begin{cases} k\lambda_P + \frac{1}{3}\Delta & \text{if} \quad k\lambda_P - \frac{1}{3}\Delta > 0, \\ \Delta - k\lambda_P & \quad otherwise, \end{cases} \\
f_{LS} &= \begin{cases} k\lambda_P - \frac{1}{3}\Delta & \text{if} \quad k\lambda_P - \frac{1}{3}\Delta > 0, \\ 0 & \quad otherwise, \end{cases}
\end{aligned}
$$

with $\Delta = (R_H - R_L)$. The optimal quantities invested in each fund are therefore equal to

$$
\begin{aligned}
q^*_{HS} &= \begin{cases} \frac{1}{2}\lambda_P + \frac{1}{6k}\Delta & \text{if} \quad k\lambda_P - \frac{1}{3}\Delta > 0, \\ \lambda_P & \quad otherwise, \end{cases} \\
q^*_{LS} &= \begin{cases} \frac{1}{2}\lambda_P - \frac{1}{6k}\Delta & \text{if} \quad k\lambda_P - \frac{1}{3}\Delta > 0, \\ 0 & \quad otherwise. \end{cases}
\end{aligned}
$$

Notice that for this to be an equilibrium we need to have that the utility of the marginal investor x is higher or equal to zero, i.e. $U_\varphi = R_\varphi - f_\varphi + (u_0 - kx) \geq 0$. We therefore need $k \leq \frac{1}{3\lambda_P}(2u_0 + R_L + R_H)$ in order for this to be a possible solution.

Case II $\mu_{HS} = \mu_{LS} = 0$, $\eta \neq 0$

The Kuhn-Tucker conditions (1) imply that $x + y = \lambda_P$, $\eta > 0$, $U_{HS} \geq 0, U_{LS} \geq 0$ and

$$
\begin{aligned}
x + \frac{\partial x}{\partial f_G}(f_{HS} - \eta) &= 0, \\
y + \frac{\partial y}{\partial f_B}(f_{LS} - \eta) &= 0.
\end{aligned}
$$

Since $x + y = \lambda_P$, it implies that $U_{HS} = U_{LS}$ and therefore x is as defined in (3) and therefore

$$
\begin{aligned}
x + \left(-\frac{1}{2k}\right)(f_{HS} - \eta) &= 0, \\
y + \left(-\frac{1}{2k}\right)(f_{LS} - \eta) &= 0.
\end{aligned}
$$

Consequently, we have that

$$
\begin{aligned}
x &= \frac{1}{2k}\left(f_{HS}-\eta\right), \\
y &= -\frac{1}{2k}\left(f_{LS}-\eta\right).
\end{aligned}
$$

Since $\eta > 0$ the profit of the HS fund manager is $f_{HS}x = \frac{f_{HS}}{2k}\left(f_{HS}-\eta\right) < \frac{\left(f_{HS}\right)^2}{2k}$ and in this case we do not obtain a global optimum.

Case III $\mu_{HS} \neq 0$, $\mu_{LS} = 0$, $\eta \neq 0$

This implies that $x + y = \lambda_P,\ \eta > 0,\ U_{HS} = 0, \mu_{HS} > 0$ and $U_{LS} \geq 0$. However, since $x + y = \lambda_P$, we have that

$$
x = \frac{R_H - f_{HS} + u_0}{k},
$$

and

$$
\begin{aligned}
\frac{\partial \mathcal{L}_{HS}}{\partial f_{HS}} &= x + f_{HS}\frac{\partial x}{\partial f_{HS}} - \mu_{HS}\left(k\frac{\partial x}{\partial f_{HS}} + 1\right) - \eta\frac{\partial x}{\partial f_{HS}} = 0, \\
\frac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} &= y + f_{LS}\frac{\partial y}{\partial f_{LS}} - \eta\frac{\partial y}{\partial f_{LS}} = 0.
\end{aligned}
$$

Since $y = \lambda_P - x$, and $\frac{\partial x}{\partial f_{LS}} = 0$ we can rewrite $\frac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} = \lambda_P - x - f_{LS}\frac{\partial x}{\partial f_{LS}} + \eta\frac{\partial x}{\partial f_{LS}} = \lambda_P - x$. This implies that $x = \lambda_P,\ y = 0$ and $f_{HS} = R_H + u_0 - k\lambda_P$. Finally, we have that $x + \left(f_{HS} - \eta\right)\frac{\partial x}{\partial f_{HS}} = 0 \Leftrightarrow \lambda_P + \left(f_{HS}-\eta\right)\left(-\frac{1}{k}\right) = 0 \Leftrightarrow k\lambda_P + \eta = f_{HS} \Leftrightarrow \eta = f_{HS} - k\lambda_P$. Therefore $\eta = R_H + u_0 - k\lambda_P - k\lambda_P = R_H + u_0 - 2k\lambda_P > 0$ if $2k\lambda_P < R_H + u_0$. In this case the market is covered by the HS fund.

Case IV $\mu_{HS} = 0$, $\mu_{LS} \neq 0$, $\eta \neq 0$

This implies that $x + y = \lambda_P,\ \eta > 0,\ U_{LS} = 0, \mu_{LS} > 0$ and $U_{HS} \geq 0$. However, since $x + y = \lambda_P$ it implies also $U_{HS} = 0$. So, we have that

$$
\begin{aligned}
x &= \frac{R_H - f_{HS} + u_0}{k}, \\
y &= \frac{R_L - f_{LS} + u_0}{k},
\end{aligned}
$$

and the conditions (2) become in this case

$$\begin{aligned}\frac{\partial \mathcal{L}_{HS}}{\partial f_{HS}} &= x + f_{HS}\frac{\partial x}{\partial f_{HS}} - \eta \frac{\partial x}{\partial f_{HS}} = 0,\\ \frac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} &= y + f_{LS}\frac{\partial y}{\partial f_{LS}} - \mu_{LS}\left(k\frac{\partial y}{\partial f_{LS}} + 1\right) - \eta \frac{\partial y}{\partial f_{LS}} = 0.\end{aligned}$$

Notice that $k\frac{\partial y}{\partial f_{LS}} + 1 = k\left(-\frac{1}{k}\right) + 1 = 0$, and therefore this case is reduced to Case 2, where we have proved that the solution obtained is not a global optimum.

Case V $\mu_{HS} = 0$, $\mu_{LS} \neq 0$, $\eta = 0$

This implies that $x + y \leq \lambda_P$, $U_{LS} = 0, \mu_{LS} > 0$ and $U_{HS} \geq 0$. Since $x + y \leq \lambda_P$, $U_{LS} = 0$ it implies also $U_{HS} = 0,$ so we have

$$\begin{aligned}x &= \frac{R_H - f_{HS} + u_0}{k},\\ y &= \frac{R_L - f_{LS} + u_0}{k},\end{aligned}$$

and therefore the conditions (2) become

$$\begin{aligned}\frac{\partial \mathcal{L}_{HS}}{\partial f_{HS}} &= x + f_{HS}\frac{\partial x}{\partial f_{HS}} = 0,\\ \frac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} &= y + f_{LS}\frac{\partial y}{\partial f_{LS}} = 0,\end{aligned}$$

since $\left(k\frac{\partial y}{\partial f_{LS}} + 1\right) = k\left(-\frac{1}{k}\right) + 1 = 0$. Consequently, we have the same solution as in Case I.1.

Case VI $\mu_{HS} \neq 0$, $\mu_{LS} = 0$, $\eta = 0$

This implies that $x + y \leq \lambda_P$, $U_{HS} = 0, \mu_{HS} > 0$ and $U_{LS} \geq 0$. Since $x + y \leq \lambda_P$, $U_{HS} = 0$ it implies also $U_{LS} = 0,$ so we have again that

$$\begin{aligned}x &= \frac{R_H - f_{HS} + u_0}{k},\\ y &= \frac{R_L - f_{LS} + u_0}{k}.\end{aligned}$$

Therefore the conditions (2) become

$$\begin{aligned}
\frac{\partial \mathcal{L}_{HS}}{\partial f_{HS}} &= x + f_{HS}\frac{\partial x}{\partial f_{HS}} = 0, \\
\frac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} &= y + f_{LS}\frac{\partial y}{\partial f_{LS}} = 0,
\end{aligned}$$

since again $k\frac{\partial x}{\partial f_{HS}} + 1 = k\left(-\frac{1}{k}\right) + 1 = 0$. Consequently, we have again the same solution as in Case I.1.

Case VII $\mu_{HS} \neq 0$, $\mu_{LS} \neq 0$, $\eta = 0$

This implies that $x + y \leq \lambda_P$, $U_{LS} = 0, \mu_{LS} > 0$ and $U_{HS} = 0\,, \mu_{LS} > 0$. Similarly to the Cases V and VI we obtain the same solution as in Case I.1.

Case VIII $\mu_{HS} \neq 0$, $\mu_{LS} \neq 0$, $\eta \neq 0$

This implies that $x + y = \lambda_P$, $U_{LS} = 0, \mu_{LS} > 0$ and $U_{HS} = 0\,, \mu_{LS} > 0$ and again we have the same solution as in Case I.1 if $\lambda_P = \frac{1}{2k}\left(2u_0 + R_H + R_L\right).$

We compare the possible equilibria to find the global optimum. The solution in Case I.1 is obtained if and only if $k \geq \frac{1}{2\lambda_P}\left(2u_0 + R_H + R_L\right)$, while the solution in Case I.2 is obtained if and only if $k \leq \frac{1}{3\lambda_P}\left(2u_0 + R_H + R_L\right)$. This implies that in the interval $k \in \left[\frac{1}{2\lambda_P}\left(2u_0 + R_H + R_L\right), \frac{1}{3\lambda_P}\left(2u_0 + R_H + R_L\right)\right]$ we have two candidates to a global optimum. To see which one is the global optimum let us compare the two profits. In case the solution is the one obtained in Case I.1, the profit of fund HS is the monopoly profit $\frac{1}{4k}\left(R_H + u_0\right)^2$ while in the case Case I.2 is equal to $\frac{1}{2k}\left(k\lambda_P + \frac{1}{3}\Delta\right)^2$. Note that $\frac{1}{4k}\left(R_H + u_0\right)^2 - \left(k\lambda_P + \frac{1}{3}\Delta\right)\left(\frac{1}{2}\lambda_P + \frac{1}{6k}\Delta\right) = -\frac{1}{36}\frac{18k^2\lambda_P^2 + 2\Delta^2 - 9\left(R_H + u_0\right)^2 + 12k\Delta\lambda_P}{k}$ is positive for all $k \in [0, k^*]$ where $k^* = -\frac{1}{6\lambda_P}\left(2\Delta - 3\sqrt{2}\left(R_H + u_0\right)\right).$

Since $\frac{1}{3\lambda_P}\left(2u_0 + R_H + R_L\right) < -\frac{1}{6\lambda_P}\left(2\Delta - 3\sqrt{2}\left(R_H + u_0\right)\right) = k^*$ this implies that for all $k \in \left[\frac{1}{2\lambda_P}\left(2u_0 + R_H + R_L\right), \frac{1}{3\lambda_P}\left(2u_0 + R_H + R_L\right)\right]$ the local monopoly is the global equilibrium. Therefore, if $\lambda_P \geq \frac{1}{2k}\left(2u_0 + R_H + R_L\right)$ the solution is the one obtained in Case I.1, otherwise the solution is the one from Case I.2. We have defined $k_1 \equiv \frac{\Delta}{3\lambda_P}$ and

$k_2 \equiv \frac{1}{2\lambda_P}(2u_0 + R_H + R_L)$. With these notations, the equilibrium fees are

$$f^*_{HS} = \begin{cases} \Delta - k\lambda_P & if \quad k \leq k_1, \\ k\lambda_P + \frac{1}{3}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2}(R_H + u_0) & if \quad k \geq k_2, \end{cases} \quad \text{and}$$

$$f^*_{LS} = \begin{cases} 0 & if \quad k \leq k_1, \\ k\lambda_P - \frac{1}{3}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2}(R_L + u_0) & if \quad k \geq k_2. \end{cases}$$

And the optimal quantities invested in each ESG fund are therefore equal to

$$q^*_{HS} = \begin{cases} \lambda_P & if \quad k \leq k_1, \\ \frac{1}{2}\lambda_P + \frac{1}{6k}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2k}(R_H + u_0) & if \quad k \geq k_2, \end{cases}$$

$$q^*_{LS} = \begin{cases} 0 & if \quad k \leq k_1, \\ \frac{1}{2}\lambda_P + \frac{1}{6k}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2k}(R_L + u_0) & if \quad k \geq k_2. \end{cases}$$

Finally, we have to determine the optimal fee for the HC fund. The optimal fee is determined in such a way that the HS fund does not want to mimic the fee of the HC fund. We find next the fee f_{HC} in each of the three cases above, which are determined by the value of k.

Case A First we considered the case when $k \leq k_1$, when the fund HS covers all the market of ESG investors and fund LS does not participate in the market (sets a fee $f_{LS} = 0$).

The fee f that makes the manager of the HS fund to be indifferent between operating only with the ESG investors and operating with both ESG and neutral investors is such that

$$f_{HS}q_{HS} = f\left(q_{HS}(f, f_{LS}) + \lambda_N\right). \tag{4}$$

Replacing the optimal values for f_{HS}, f_{LS} and q_{HS} in this case we obtain that (4) is equivalent to

$$(\Delta - k\lambda_P)\,\lambda_P = f(\lambda_P + \lambda_N).$$

Therefore the manager of the HC fund chooses

$$f^*_{HC} = \min\left\{\Delta, \frac{\lambda_P}{\lambda_N + \lambda_P}\,(\Delta - k\lambda_P)\right\} - \varepsilon\; = \frac{\lambda_P}{\lambda_N + \lambda_P}\,(\Delta - k\lambda_P) - \varepsilon,$$

with ε positive and small enough, and at this fee the manager of the fund HC does not have incentives to deviate.

Case B In the second case we consider values for k such that $k_1 \leq k \leq k_2$. In this case the market is covered by the two funds. The fee f that makes the manager of the HC fund to be indifferent between operating only with the ESG investors and operating with both ESG and neutral investors is such that

$$f_{HS}q_{HS} = f\,(q_{HS}\,(f, f_{LS}) + \lambda_N)\,. \tag{5}$$

We define $T \equiv f_{HS} = k\lambda_P + \frac{1}{3}\Delta$. With these notations the equation (5) can be rewritten as

$$\frac{T^2}{2k} = f\left(\frac{T}{k} - \frac{f}{2k} + \lambda_N\right).$$

Notice that in order to be able to serve all the market of neutral investors, the manager of the HSfund should deviate to a fee $f < f_{HS} = T$.

We define

$$G\,(f) \equiv \frac{T^2}{2k} - f\left(\frac{T}{k} - \frac{f}{2k} + \lambda_N\right)$$

and it results that

$$G\,(f) = \frac{1}{18}\frac{9k^2\lambda_P^2 - 6f\Delta + \Delta^2 + 9f^2 - 18fk\lambda_N - 18fk\lambda_P + 6k\Delta\lambda_P}{k}.$$

Notice that $G'\,(f) = 0$ has a solution $\widetilde{f} = \frac{1}{3}\Delta + k\lambda_N + k\lambda_P > k\lambda_P + \frac{1}{3}\Delta$ and this implies that for all $f \in \left(0, \widetilde{f}\right)$ the function $G\,(f)$ is decreasing. Moreover, since $G\,(0) = \frac{1}{18}\frac{(\Delta + 3k\lambda_P)^2}{k} > 0$ and $G\left(k\lambda_P + \frac{1}{3}\Delta\right) = -\frac{1}{3}\lambda_N\,(\Delta + 3k\lambda_P) < 0$ it implies that it exists a fee $f^* \in \left(0, \widetilde{f}\right)$ such that $G\,(f) = 0$. Since we have that $f^* < k\lambda_P + \frac{1}{3}\Delta$, the manager of the

HC fund chooses $f^*_{HC} = \min\{f^* - \varepsilon, \Delta\}$ with ε positive and small enough.

Case C Finally, we consider the case when the market is not covered, i.e. when $k \geq k_2$. As explained above, in this case the fund HS acts as a monopoly.

The fee f that makes the manager of the HS fund to be indifferent between operating only with the ESG investors and operating with both ESG and neutral investors is such that

$$f_{HS} q_{HS} = f\left(q_{HS}(f) + \lambda_N\right), \tag{6}$$

where $f_{HS} = \dfrac{1}{2}\left(R_H + u_0\right)$ and $q_{HS} = \dfrac{(R_H + u_0)}{2k}$. Replacing these values (6) becomes

$$\frac{\left(R_H + u_0\right)^2}{4k} = f\left(\frac{R_H - f + u_0}{k} + \lambda_N\right).$$

We define

$$H(f) \equiv -4f\left(R_H + u_0\right) + \left(R_H + u_0\right)^2 + 4f^2 - 4fk\lambda_N.$$

Notice that the equation $H'(f) = 0$ has a solution $\widehat{f} = \frac{1}{2}\left(R_H + u_0\right) + \frac{1}{2}k\lambda_N$. This implies that for all fees $0 < f < \widehat{f}$ we have that $H'(f) < 0$. Since $H(0) = \left(R_H + u_0\right)^2 > 0$ and $H\left(\dfrac{1}{2}\left(R_H + u_0\right)\right) = -\frac{1}{2}\left(R_H + u_0\right)\left(5\left(R_H + u_0\right) + 9k\lambda_N\right) < 0$, there exists a fee $f^{**} \in \left(0, \widehat{f}\right)$ such that $H(f^{**}) = 0$. As a result the fund HC manager should set a fee $f^*_{HC} = \min\{\Delta, f^{**}\} - \varepsilon$.

To summarize, we obtain that, the optimal fee for the HC fund in this case is

$$f^*_{HC} = \begin{cases} \dfrac{\lambda_P}{\lambda_N + \lambda_P}\left(\Delta - k\lambda_P\right) - \varepsilon & \text{if} \quad k \leq k_1, \\ \min\{\Delta, f^*\} - \varepsilon & \text{if} \quad k \in (k_1, k_2), \\ \min\{\Delta, f^{**}\} - \varepsilon & \text{if} \quad if\ k \geq k_2. \end{cases}$$

Notice that, in equilibrium, in order for fund HS not to be able to mimic fund HC we have $f^*_{HC} < f_{HS}$ always. □

Proof of Proposition 2. Similarly to the case when both conventional funds and ESG exist in the market, we write the maximization problems for fund HS and LS :

$$\begin{aligned}
\max_{f_{HS}} \Pi_S &= f_{HS} q_{HS} \\
s.t.\ U_{HS} &= R_H - f_{HS} + u_0 - kx \geq 0 \\
x + y &\leq \lambda_{LS} \\
f_{HS} &\geq 0 \\
x &\geq 0
\end{aligned}$$

and

$$\begin{aligned}
\max_{f_{LS}} \Pi_{HS} &= f_{LS} q_{LS} \\
s.t.\ U_{LS} &= R_L - f_{LS} + u_0 - ky \geq 0 \\
x + y &\leq \lambda_{LS} \\
f_{HS} &\geq 0 \\
y &\geq 0.
\end{aligned}$$

The only difference is that now the neutral investors may invest with any of the funds HS and LS depending on which one gives a higher net return. Thus, the demand for the fund HS is

$$q_{HS}(f_{HS}, f_{LS}) = \begin{cases} x + \lambda_N & \text{if} \quad f_{HS} < \Delta + f_{LS}, \\ x + \dfrac{\lambda_N}{2} & \text{if} \quad f_{HS} = \Delta + f_{LS}, \\ x & \text{if} \quad f_{HS} > \Delta + f_{LS}, \end{cases}$$

and the demand for fund LS is

$$q_{LS}(f_{HS}, f_{LS}) = \begin{cases} \lambda_P - x & \text{if} \quad f_{HS} < \Delta + f_{LS}, \\ \lambda_P - x + \dfrac{\lambda_N}{2} & \text{if} \quad f_{HS} = \Delta + f_{LS}, \\ \lambda_P - x + \lambda_N & \text{if} \quad f_{HS} > \Delta + f_{LS}. \end{cases}$$

The Lagrangean for the problem of the manager of the HS fund is

$$\mathcal{L}_{HS} = f_{HS} q_{HS} - \mu_{HS} (-u_0 + kx - R_H + f_{HS}) - \eta (x + y - \lambda_P) + \gamma x,$$

and the one for the problem of the manager of the LS fund is

$$\mathcal{L}_{LS} = f_{LS}q_{LS} - \mu_{LS}\left(-u_0 + ky - R_L + f_{LS}\right) - \eta\left(x + y - \lambda_P\right) + \delta y.$$

The Kuhn-Tucker conditions for these problems are

$$\begin{aligned} \frac{\partial \mathcal{L}_\varphi}{\partial f_\varphi} &\leq 0, \quad f_\varphi \geq 0, \; f_\varphi \frac{\partial \mathcal{L}_\varphi}{\partial f_\varphi} = 0, \\ U_\varphi &\geq 0, \; \mu_\varphi \geq 0 \;, \; \mu_\varphi U_\varphi = 0, \\ x + y &\leq \lambda_U, \; \eta \geq 0, \; \eta\left(x + y - \lambda_U\right) = 0 \end{aligned} \tag{7}$$

for $\varphi \in \{HS, LS\}$. Using the Kuhn-Tucker conditions and similarly to the case when both ESG funds and conventional funds co-existing in the market, we have three candidates to the global optimum: when both funds act as local monopolies, when they cover the market of ESG investors and when the market of ESG investors is covered by the HS fund and the fund LS is driven out of the market. Moreover, the existence of a positive mass of neutral investors in search for the highest possible net return creates three cases depending on the relationship of the net returns of the funds HS and LS.

When the firms act as local monopolies the revenue for the funds HS and LS equal to

$$\Pi^M_{HS}\left(f_{HS}, f_{LS}\right) = \begin{cases} \left(\dfrac{R_H - f_{HS} + u_0}{k} + \lambda_N\right) f_{HS} & \text{if} \quad R_H - f_{HS} > R_L - f_{LS}, \\ \left(\dfrac{R_H - f_{HS} + u_0}{k} + \dfrac{\lambda_N}{2}\right) f_{HS} & \text{if} \quad R_H - f_{HS} = R_L - f_{LS}, \\ \dfrac{R_H - f_{HS} + u_0}{k} f_{HS} & if \quad R_H - f_{HS} < R_L - f_{LS}, \end{cases}$$

$$\Pi^M_{LS}\left(f_{HS}, f_{LS}\right) = \begin{cases} \dfrac{R_L - f_{LS} + u_0}{k} f_{LS} & \text{if} \quad R_H - f_{HS} > R_L - f_{LS}, \\ \left(\dfrac{R_L - f_{LS} + u_0}{k} + \dfrac{\lambda_N}{2}\right) f_{LS} & \text{if} \quad R_H - f_{HS} = R_L - f_{LS}, \\ \left(\dfrac{R_L - f_{LS} + u_0}{k} + \lambda_N\right) f_{LS} & if \quad R_H - f_{HS} < R_L - f_{LS}, \end{cases}$$

respectively.

Case 1 $R_H - f_{HS} > R_L - f_{LS}$

In this case the demand for fund HS equals $\dfrac{R_H - f_{HS} + u_0}{k} + \lambda_N$ and the demand for the fund LS equals $\dfrac{R_L - f_{LS} + u_0}{k}$. Therefore the best responses functions are

$$\begin{aligned}
\phi_{HS}^{M1}(f_{LS}) &= \frac{1}{2}(R_H + u_0 + k\lambda_N), \\
\phi_{LS}^{M1}(f_{HS}) &= \frac{1}{2}(R_L + u_0).
\end{aligned}$$

Case 2 $R_H - f_{HS} = R_L - f_{LS}$

In this case the demand for fund HS equals $\dfrac{R_H - f_{HS} + u_0}{k} + \dfrac{\lambda_N}{2}$ and the demand for the fund LS equals $\dfrac{R_L - f_{LS}}{k} + \dfrac{\lambda_N}{2}$, and therefore the best responses are

$$\begin{aligned}
\phi_{HS}^{M2}(f_{LS}) &= \frac{1}{2}\left(R_H + u_0 + \frac{1}{2}k\lambda_N\right) \\
\phi_{LS}^{M2}(f_{HS}) &= \frac{1}{2}\left(R_L + u_0 + \frac{1}{2}k\lambda_N\right).
\end{aligned}$$

In this case $R_H - f_{HS} = \frac{1}{2}R_H - u_0 - \frac{1}{4}k\lambda_N > R_L - f_{LS} = \frac{1}{2}R_L - u_0 - \frac{1}{4}k\lambda_N$ since $R_H > R_{LS}$ and therefore, in this case we do not have an equilibrium.

Case 3 $R_H - f_{HS} < R_L - f_{LS}$

In this case the demand for fund HS equals $\dfrac{R_H - f_{HS} + u_0}{k}$ and the demand for the fund LS equals $\dfrac{R_L - f_{LS} + u_0}{k} + \lambda_N$, and therefore the best responses are

$$\begin{aligned}
\phi_{HS}^{M3}(f_{LS}) &= \frac{1}{2}(R_H + u_0), \\
\phi_{LS}^{M3}(f_{HS}) &= \frac{1}{2}(R_H + u_0 + k\lambda_N).
\end{aligned}$$

As a result, the best responses in case the market is not covered are:

$$\phi_{HS}^{M}(f_{LS}) = \begin{cases} \phi_{HS}^{M1}(f_{LS}) = \frac{1}{2}(R_H + u_0 + k\lambda_N) & \text{if} \quad R_H - f_{HS} > R_L - f_{LS}, \\ \phi_{HS}^{M2}(f_{LS}) = \frac{1}{2}(R_H + u_0) & \text{if} \quad R_H - f_{HS} < R_L - f_{LS}, \end{cases}$$

$$\phi_{LS}^{M}(f_{HS}) = \begin{cases} \phi_{LS}^{M1}(f_{HS}) = \frac{1}{2}(R_L + u_0) & \text{if} \quad R_H - f_{HS} > R_L - f_{LS}, \\ \phi_{LS}^{M2}(f_{HS}) = \frac{1}{2}(R_L + u_0 + k\lambda_N) & \text{if} \quad R_H - f_{HS} < R_L - f_{LS}. \end{cases}$$

The equilibrium quantities in this case are

$$x\left(f_{HS}, f_{LS}\right) = \begin{cases} \dfrac{1}{2k}\left(R_H + u_0 - k\lambda_N\right) & \text{if} \quad R_H - f_{HS} > R_L - f_{LS}, \\ \dfrac{1}{2k}\left(R_H + u_0\right) & \text{if} \quad R_H - f_{HS} < R_L - f_{LS}, \end{cases}$$

and

$$y\left(f_{HS}, f_{LS}\right) = \begin{cases} \dfrac{1}{2k}\left(R_L + u_0\right) & \text{if} \quad R_H - f_{HS} > R_L - f_{LS}, \\ \dfrac{1}{2k}\left(R_L + u_0 - k\lambda_N\right) & \text{if} \quad R_H - f_{HS} < R_L - f_{LS}. \end{cases}$$

Therefore, we have two candidates to equilibrium:
$\left(f^*_{HS}, f^*_{LS}\right) \in \left\{ \left(\frac{1}{2}\left(R_H + u_0 + k\lambda_N\right), \frac{1}{2}\left(R_L + u_0\right)\right), \left(\frac{1}{2}\left(R_H + u_0\right), \frac{1}{2}\left(R_L + u_0 + k\lambda_N\right)\right) \right\}$.

Let us consider first $\left(f^*_{HS}, f^*_{LS}\right) = \left(\frac{1}{2}\left(R_H + u_0 + k\lambda_N\right), \frac{1}{2}\left(R_L + u_0\right)\right)$.

Since $\phi^{M1b}_{HS}\left(f_{LS}\right) = \frac{1}{2}\left(R_H + u_0 + k\lambda_N\right) > \phi^{M2}_{HS}\left(f_{LS}\right) = \frac{1}{2}\left(R_H + u_0\right)$ whenever $\lambda_N > 0$, the fund HS maximizes the revenue. Notice that this equilibrium satisfies the condition $R_H - f_{HS} > R_L - f_{LS}$.

We need to check whether the fund LS is willing to deviate from setting $\frac{1}{2}R_L$. As explained, the only option to deviate would be to set $f_{LS} = \frac{1}{2}R_L + \frac{1}{2}k\lambda_N$. The fund LS's revenue is in this case equal to

$$\Pi^{M1}_{LS}\left(\phi^{M1}_{LS}\left(f_{HS}\right), f_{HS}\right) = \frac{1}{4k}\left(R_L + u_0\right)^2.$$

And if it deviates to $\phi^{M2}_{LS}\left(f_{HS}\right) = \frac{1}{2}\left(R_L + u_0 + k\lambda_N\right)$ it equals

$$\Pi^{M1}_{LS}\left(\phi^{M2}_{LS}\left(f_{HS}\right), f_{HS}\right) = \left(\frac{R_L - \frac{1}{2}\left(R_L + u_0 + k\lambda_N\right)}{k}\right)\frac{1}{2}\left(R_L + u_0 + k\lambda_N\right) < \frac{R_L}{4k}.$$

Notice that in this case we have that $R_H - f_{HS} = \frac{1}{2}\left(R_H - u_0 - k\lambda_N\right) >$
$\frac{1}{2}\left(R_L - u_0 - k\lambda_N\right) = R_L - \phi^{M2}_{LS}\left(f_{HS}\right)$ so despite deviating the fund LS does not manage to attract the neutral investors and therefore is not an equilibrium.

The equilibrium $\left(f^*_{HS}, f^*_{LS}\right) = \left(\frac{1}{2}\left(R_H + u_0 + k\lambda_N\right), \frac{1}{2}\left(R_L + u_0\right)\right)$ satisfies the condition $R_H - f_{HS} > R_L - f_{LS}$ if and only if $\frac{1}{2}\left(R_H - u_0 - k\lambda_N\right) > \frac{1}{2}\left(R_L - u_0\right)$ or equivalently $k < \frac{\Delta}{\lambda_N}$.

Moreover, this equilibrium with local monopolies exists if and only if $x+y<\lambda_P$, which is equivalent to $\frac{1}{2k}\left(R_H+u_0-k\lambda_N\right)+\frac{1}{2k}\left(R_H+u_0\right)<\lambda_P$ or $\frac{2u_0+R_H+R_L}{2\lambda_P+\lambda_N}<k$. We define k_2^O as $k_2^O\equiv\frac{2u_0+R_H+R_L}{2\lambda_P+\lambda_N}$.

When $k>\frac{\Delta}{\lambda_N}$ we have that $R_H-f_{HS}=\frac{1}{2}\left(R_H-k\lambda_N\right)<\frac{1}{2}\left(R_H-\Delta\right)=\frac{1}{2}R_L=R_L-f_{LS}$, so this cannot be an equilibrium. In this case, in order to attract the neutral investors, the funds start to undercut each other until the fund LS is driven out of the market $f_{LS}=0$. However, the fund LS would prefer to charge a higher fee $f_{LS}=\frac{1}{2}R_L$ and make a positive revenue serving the ESG investors close to him. Therefore, no equilibrium exists in this case.

The other candidate equilibrium is $\left(f_{HS}^*,f_{LS}^*\right)=\left(\frac{1}{2}\left(R_H+u_0\right),\frac{1}{2}\left(R_L+u_0+k\lambda_N\right)\right)$ when $R_H-f_{HS}<R_L-f_{LS}$. However, notice that $R_H-f_{HS}=\frac{1}{2}\left(R_H-u_0\right)>\frac{1}{2}\left(R_L-u_0\right)>\frac{1}{2}\left(R_L-u_0-k\lambda_N\right)=R_L-f_{LS}$, and therefore this is never an equilibrium.

Consequently, the equilibrium $\left(f_{HS}^*,f_{LS}^*\right)=\left(\frac{1}{2}R_H+\frac{1}{2}k\lambda_N,\frac{1}{2}R_L\right)$ exists if $k<\frac{\Delta}{\lambda_N}$ and is unique.

Let us consider next the case when $k<\frac{2u_0+R_H+R_L}{2\lambda_P+\lambda_N}$. In this case the market is covered. Similarly, to the case where both conventional and ESG funds coexist, the location of the marginal ESG investor is

$$x=\frac{\lambda_P}{2}+\frac{R_H-f_{HS}-R_L+f_{LS}}{2k}=\frac{\lambda_P}{2}+\frac{r_{HS}-r_{LS}}{2k}.$$

It follows that

$$\begin{aligned} x &= \frac{\lambda_P}{2}+\frac{r_{HS}-r_{LS}}{2k}, \\ y &= \frac{\lambda_P}{2}-\frac{r_{HS}-r_{LS}}{2k}. \end{aligned}$$

The optimal demand for fund HS is

$$q_{HS}\left(f_{HS},f_{LS}\right)=\begin{cases} \frac{\lambda_P}{2}+\frac{R_H-f_{HS}-R_L+f_{LS}}{2k}+\lambda_N & \text{if } \quad R_H-f_{HS}>R_L-f_{LS}, \\ \frac{\lambda_P}{2}+\frac{R_H-f_{HS}-R_L+f_{LS}}{2k}+\frac{\lambda_N}{2} & \text{if } \quad R_H-f_{HS}=R_L-f_{LS}, \\ \frac{\lambda_P}{2}+\frac{R_H-f_{HS}-R_L+f_{LS}}{2k} & \text{if } \quad R_H-f_{HS}<R_L-f_{LS}. \end{cases}$$

Similarly, the optimal demand for fund LS is

$$d_{LS}(f_{HS}, f_{LS}) = \begin{cases} \dfrac{\lambda_P}{2} - \dfrac{R_H - f_{HS} - R_L + f_{LS}}{2k} & \text{if} \quad R_H - f_{HS} > R_L - f_{LS}, \\ \dfrac{\lambda_P}{2} - \dfrac{R_H - f_{HS} - R_L + f_{LS}}{2k} + \dfrac{\lambda_N}{2} & \text{if} \quad R_H - f_{HS} = R_L - f_{LS}, \\ \dfrac{\lambda_P}{2} + \dfrac{R_H - f_{HS} - R_L + f_{LS}}{2k} + \lambda_N & \text{if} \quad R_H - f_{HS} < R_L - f_{LS}, \end{cases}$$

and the revenues are

$$\Pi_{HS}(f_{HS}, f_{LS}) = \begin{cases} \left(\dfrac{\lambda_P}{2} + \dfrac{R_H - f_{HS} - R_L + f_{LS}}{2k} + \lambda_N\right) f_{HS} & \text{if} \quad R_H - f_{HS} > R_L - f_{LS}, \\ \left(\dfrac{\lambda_P}{2} + \dfrac{R_H - f_{HS} - R_L + f_{LS}}{2k} + \dfrac{\lambda_N}{2}\right) f_{HS} & \text{if} \quad R_H - f_{HS} = R_L - f_{LS}, \\ \left(\dfrac{\lambda_P}{2} + \dfrac{R_H - f_{HS} - R_L + f_{LS}}{2k}\right) f_{HS} & \text{if} \quad R_H - f_{HS} < R_L - f_{LS}, \end{cases}$$

$$\Pi_{LS}(f_{HS}, f_{LS}) = \begin{cases} \left(\dfrac{\lambda_P}{2} - \dfrac{R_H - f_{HS} - R_L + f_{LS}}{2k}\right) f_{LS} & \text{if} \quad R_H - f_{HS} > R_L - f_{LS}, \\ \left(\dfrac{\lambda_P}{2} - \dfrac{R_H - f_{HS} - R_L + f_{LS}}{2k} + \dfrac{\lambda_N}{2}\right) f_{LS} & \text{if} \quad R_H - f_{HS} = R_L - f_{LS}, \\ \left(\dfrac{\lambda_P}{2} - \dfrac{R_H - f_{HS} - R_L + f_{LS}}{2k} + \lambda_N\right) f_{LS} & \text{if} \quad R_H - f_{HS} < R_L - f_{LS}, \end{cases}$$

respectively.

Case 1 $R_H - f_{HS} > R_L - f_{LS}$

The revenue functions in this case are the following

$$\begin{aligned} \Pi^1_{HS}(f_{HS}, f_{LS}) &= \frac{1}{2k}(k\lambda_P + 2k\lambda_N + \Delta + f_{LS} - f_{HS}) f_{HS}, \\ \Pi^1_{LS}(f_{HS}, f_{LS}) &= \frac{1}{2k}(k\lambda_P - (\Delta + f_{LS} - f_{HS})) f_{LS}. \end{aligned}$$

From the first order conditions for the maximization of the revenue for each fund we obtain each fund's best reply in this case

$$\begin{aligned} \phi^1_{HS}(f_{LS}) &= \frac{1}{2}(k\lambda_P + 2k\lambda_N + \Delta + f_{LS}), \\ \phi^1_{LS}(f_{HS}) &= \frac{1}{2}(k\lambda_P - \Delta + f_{HS}). \end{aligned}$$

The equilibrium in this case is

$$
\begin{aligned}
f_{HS} &= k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta, \\
f_{LS} &= k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta.
\end{aligned}
$$

Notice that $f_{HS} > 0$, while $f_{LS} = k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta > 0$ if and only if $k > \frac{\Delta}{2k\lambda_N + 3k\lambda_P}$. Otherwise $f_{LS} = 0$ and the fund HS covers the entire market $R_H - f_{HS} + u_0 - k\lambda_P = R_L + u_0$, so $f_{HS} = \Delta - k\lambda_P$.

Case 2 $R_H - f_{HS} = R_L - f_{LS}$

Notice that in this case the first order conditions are

$$
\begin{aligned}
\frac{1}{2k}\left(k\lambda_P + \Delta + f_{LS} - f_{HS} + k\lambda_N\right) - \frac{1}{2k}f_{HS} &= 0, \\
\frac{1}{2k}\left(k\lambda_P - (\Delta + f_{LS} - f_{HS}) + k\lambda_N\right) - \frac{1}{2k}f_{LS} &= 0
\end{aligned}
$$

and the solution of this system is

$$
\begin{aligned}
f_{HS} &= k\lambda_P + k\lambda_N + \frac{1}{3}\Delta, \\
f_{LS} &= k\lambda_P + k\lambda_N - \frac{1}{3}\Delta.
\end{aligned}
$$

However, since $f_{HS} = \Delta + f_{LS}$ this can be a solution only if $\Delta = 0$ (as $2k\lambda_P + k\lambda_N + \frac{1}{3}\Delta = \Delta + 2k\lambda_P + k\lambda_N - \frac{1}{3}\Delta$). Moreover, notice that this cannot be an equilibrium because by undercutting by an ε any fund can be better off by increasing her demand with the entire mass of neutral investors, λ_P.

Case 3 $R_H - f_{HS} < R_L - f_{LS}$

The revenue functions in this case are the following

$$
\begin{aligned}
\Pi^3_{HS}\left(f_{HS}, f_{LS}\right) &= \frac{1}{2k}\left(\Delta + f_{LS} - f_{HS} + k\lambda_P\right) f_{HS}, \\
\Pi^3_{LS}\left(f_{HS}, f_{LS}\right) &= \frac{1}{2k}\left(k\lambda_P - (\Delta + f_{LS} - f_{HS}) + 2k\lambda_N\right) f_{LS}.
\end{aligned}
$$

From the first order conditions for the maximization of the revenue for each fund we obtain each fund's best reply in this case equal to

$$\begin{aligned}
\phi^3_{HS}(f_{LS}) &= \frac{1}{2}(k\lambda_P + \Delta + f_{LS}),\\
\phi^3_{LS}(f_{HS}) &= \frac{1}{2}(k\lambda_P + 2k\lambda_N - \Delta + f_{HS}),
\end{aligned}$$

and the equilibrium fees are:

$$\begin{aligned}
f_{HS} &= k\lambda_P + \frac{2}{3}k\lambda_N + \frac{1}{3}\Delta,\\
f_{LS} &= k\lambda_P + \frac{4}{3}k\lambda_N - \frac{1}{3}\Delta.
\end{aligned}$$

As a result, the best responses in case the market is covered are:

$$\phi_{HS}(f_{LS}) = \begin{cases} \phi^1_{HS}(f_{LS}) = \frac{1}{2}(k\lambda_P + 2k\lambda_N + \Delta + f_{LS}) & \text{if } R_H - f_{HS} > R_L - f_{LS},\\ \phi^2_{HS}(f_{LS}) = \frac{1}{2}(k\lambda_P + \Delta + f_{LS}) & \text{if } R_H - f_{HS} < R_L - f_{LS}, \end{cases}$$

$$\phi_{LS}(f_{HS}) = \begin{cases} \phi^1_{LS}(f_{HS}) = \tfrac{1}{2}(k\lambda_P - \Delta + f_{HS}) & \text{if } R_H - f_{HS} > R_L - f_{LS},\\ \phi^2_{LS}(f_{HS}) = \tfrac{1}{2}(2k\lambda_N + k\lambda_P - \Delta + f_{HS}) & \text{if } R_H - f_{HS} < R_L - f_{LS}. \end{cases}$$

Notice that when $2k\lambda_N = 0$ then $\phi^1_{HS}(f_{LS}) = \phi^2_{HS}(f_E)$, so the best response is linear. When $2k\lambda_N > 0$ there is a jump in the best responses of the HS fund. Notice also $2k\lambda_N \geq 0$, it follows that $\phi^1_{HS}(f_{LS}) \geq \phi^2_{HS}(f_{LS})$ and $\phi^2_{LS}(f_{HS}) \geq \phi^1_{LS}(f_{HS})$.

Therefore, we have two candidates to equilibrium: $(f_{HS}, f_{LS}) \in \left\{\left(k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta,\right.\right.$ $\left.\left. k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta\right), \left(k\lambda_P + \frac{2}{3}k\lambda_N + \frac{1}{3}\Delta, k\lambda_P + \frac{4}{3}k\lambda_N - \frac{1}{3}\Delta\right)\right.$.

Let us consider first $(f_{HS}, f_{LS}) = \left(k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta,\ k\lambda_P + \frac{1}{3}k\lambda_N - \frac{1}{3}\Delta\right)$. Since $\phi^1_{HS}(f_{LS}) \geq \phi^2_{HS}f_{LS}$, the fund HS maximizes the revenue. Notice that this equilibrium satisfies the condition $R_H - f_{HS} > R_L - f_{LS}$ if and only if $\Delta > 2k\lambda_N$.

We need to check whether the fund LS is willing to deviate from setting $f_{LS} = k\lambda_P + \frac{1}{3}k\lambda_N - \frac{1}{3}\Delta$ when the fund HS sets $k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta$. As explained, the only option to deviate would be to set $f_{LS} = k\lambda_P + \frac{4}{3}k\lambda_N - \frac{1}{3}\Delta$.

Let us check whether this is an equilibrium. The revenue functions in this case are:

$$\begin{aligned}
\Pi^1_{LS}\left(f_{HS}, \phi^1_{LS}(f_{HS})\right) &= \left(\frac{\lambda_P}{2} - \frac{R_H - f_{HS} - R_L + f_{LS}}{2k}\right) f_{LS} \\
&= \frac{1}{2k}\left(k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta\right)^2, \\
\Pi^1_{HS}\left(f_{HS}, \phi^2_{LS}(f_{HS})\right) &= \left(\frac{\lambda_P}{2} - \frac{1}{6}\frac{\Delta}{k}\right)\left(k\lambda_P + \frac{4}{3}k\lambda_N - \frac{1}{3}\Delta\right).
\end{aligned}$$

Since $\Pi^1_{HS}\left(f_{HS}, \phi^1_{LS}(f_{HS})\right) - \Pi^1_{HS}\left(f_{HS}, \phi^2_{LS}(f_{HS})\right) = \frac{2}{9}k\lambda_N^2 > 0$ the LS fund does not have any incentive to deviate and $(f_{HS}, f_{LS}) = \left(k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta,\ k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta\right)$ is an equilibrium if $\Delta > 2k\lambda_N$.

Let us consider next $(f_{HS}, f_{LS}) = \left(k\lambda_P + \frac{2}{3}k\lambda_N + \frac{1}{3}\Delta, k\lambda_P + \frac{4}{3}k\lambda_N - \frac{1}{3}\Delta\right)$. This is an equilibrium if $R_H - f_{HS} < R_L - f_{LS}$. However $R_H - f_{HS} - R_L + f_{LS} = \frac{1}{3}(\Delta + 2k\lambda_N) > 0$, so this cannot be an equilibrium.

As a result the only equilibrium is

$$\begin{aligned}
f_{HS} &= k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta, \\
f_{LS} &= k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta,
\end{aligned}$$

and $f_{LS} \geq 0$ implies $k \geq \frac{\Delta}{2\lambda_N + 3\lambda_P}$. We denote by $k_1^O \equiv \frac{\Delta}{2\lambda_N + 3\lambda_P}$, and $k_2^O \equiv \frac{2u_0 + R_H + R_L}{\lambda_N + 2\lambda_P} - \frac{\Delta}{2\lambda_N}$.

Summarizing, if $\Delta > 2k\lambda_N$ the equilibrium exists except if $k \in \left(\min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\}, k_2^O\right)$, and the equilibrium fees and quantities are:

$$f_{HS}^{O*} = \begin{cases} \Delta - k\lambda_P & \text{if} \quad k \leq k_1^O, \\ k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta & \text{if} \quad k_1^O < k \leq \min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\}, \\ \frac{1}{2}\left(R_H + u_0 + k\lambda_N\right) & \text{if} \quad k_2^O < k \leq \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}, \end{cases}$$

$$f_{LS}^{O*} = \begin{cases} 0 & \text{if} \quad k \leq k_1^O, \\ k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta & \text{if} \quad k_1^O \leq k \leq \min\left\{\frac{\Delta}{2\lambda_N}, k_2^O\right\}, \\ \frac{1}{2}\left(R_L + u_0\right) & \text{if} \quad k_2^O < k \leq \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}, \end{cases}$$

$$
q_{HS}^{O*} = \begin{cases} \lambda_P + \lambda_N & \text{if} \quad k \le k_1^O, \\ \frac{1}{2}\lambda_P + \frac{2}{3}\lambda_N + \frac{1}{6k}\Delta & \text{if} \quad k_1^O < k \le \min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\}, \\ \frac{1}{2k}\left(R_H + u_0 + k\lambda_N\right) & \text{if} \quad k_2^O < k \le \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}, \end{cases}
$$

$$
q_{LS}^{O*} = \begin{cases} 0 & \text{if} \quad k \le k_1^O, \\ \frac{1}{2}\lambda_P + \frac{1}{3}\lambda_N - \frac{1}{6k}\Delta & \text{if} \quad k_1^O < k \le \min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\}, \\ \frac{1}{2k}\left(R_L + u_0\right) & \text{if} \quad k_2^O < k \le \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}. \end{cases}
$$

If $2k\lambda_N \ge \Delta$, the net return of fund HS becomes smaller than the one from fund LS, so all the neutral investors decide to invest with the fund LS.

However, the fund HS might try to undercut fund LS such that all neutral investors invest with it. To do this chooses $f_{HS} = \Delta - \varepsilon$ such that it forces fund LS out of the market, $f_{LS} = 0$. However, the fund LS can always deviate from this equilibrium and choose a positive fee and serve locally the ESG investors with preferences similar to her investment objective. So in this case no equilibrium exists. □

Proof of Proposition 3. We compare the fees all funds charge in equilibrium. Since

$$
k_1 \equiv \frac{\Delta}{3\lambda_P} > k_1^O = \frac{\Delta}{2\lambda_N + 3\lambda_P} \text{ and}
$$
$$
k_2 \equiv \frac{2u_0 + R_H + R_L}{2\lambda_P} > k_2^O = \frac{2u_0 + R_H + R_L}{\lambda_N + 2\lambda_P}
$$

We consider the following cases:

Case 1. $k \le k_1^O$

In this case the fees are equal $f_{HS} = f_{HS}^O = \Delta - k\lambda_P$ and $f_{LS} = f_{LS}^O = 0$.

Case 2. $k_1^O < k \le k_1$

In this case the fees of the HS fund are $f_{HS} = \Delta - k\lambda_P$ and $f_{HS}^O = k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta$. Since $f_{HS} = \Delta - k\lambda_P \le k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta \le k\lambda_P + \frac{1}{3}\Delta = f_{HS}^O$, we have that $f_{HS}^O \ge f_{HS}$.

Similarly, the fees of fund LS are $f_{LS} = 0$ and $f_{LS}^O = k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta \ge 0$ and therefore $f_{LS}^O \ge f_{LS}$.

Case 3. $k_1 < k \leq k_2^O$

In this case the fees of the HS fund are $f_{HS} = k\lambda_P + \frac{1}{3}\Delta$ and $f_{HS}^O = k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta \geq f_{HS}$, respectively, whenever $\lambda_N \geq 0$. Similarly, the fees of fund LS are $f_{LS} = k\lambda_P - \frac{1}{3}\Delta$ and $f_{LS}^O = k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta \geq f_{LS}$.

Case 4. $k_2^O < k \leq k_2$

In this case the fees of the HS fund are $f_{HS} = k\lambda_P + \frac{1}{3}\Delta$ and $f_{HS}^O = \frac{1}{2}\left(R_H + u_0 + k\lambda_N\right) > f_{HS}$. To see this one can see that $\frac{1}{2}\left(R_H + u_0 + k\lambda_N\right) \geq k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta$ for all $k \geq \frac{3u_0 + 2R_L + R_H}{5\lambda_N + 6\lambda_P}$. But since $\frac{3u_0 + 2R_L + R_H}{5\lambda_N + 6\lambda_P} < \frac{2u_0 + R_H + R_L}{\lambda_N + 2\lambda_P}$, it implies that this condition is satisfied always in this particular case. Moreover since $k\lambda_P + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta > k\lambda_P + \frac{1}{3}\Delta = f_{HS}$, it implies that $f_{HS}^O > f_{HS}$.

Similarly, the fees of the LS fund are $f_{LS} = k\lambda_P - \frac{1}{3}\Delta$ and $f_{LS}^O = \frac{1}{2}\left(R_L + u_0\right)$. In a similar manner we show that $f_{LS}^O = \frac{1}{2}\left(R_L + u_0\right) \geq k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta$ for all $k \leq \frac{3u_0 + R_L + 2R_H}{4\lambda_N + 6\lambda_P}$. Since $\frac{3u_0 + R_L + 2R_H}{4\lambda_N + 6\lambda_P} \leq \frac{2u_0 + R_H + R_L}{\lambda_N + 2\lambda_P}$ it implies that the relationship is true for all the parameters in this case and therefore we have that $f_{LS}^O = \frac{1}{2}\left(R_L + u_0\right) \geq k\lambda_P + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta \geq k\lambda_P - \frac{1}{3}\Delta = f_{LS}$.

Case 5. $k_2 < k \leq k_2^O$

In this case the fees of the HS fund are $f_{HS} = \frac{1}{2}\left(R_H + u_0\right)$ and $f_{HS}^O = \frac{1}{2}\left(R_H + u_0 + k\lambda_N\right) \geq f_{HS}$. The fees of fund LS are $f_{LS} = f_{LS}^O = \frac{1}{2}\left(R_L + u_0\right)$.

Consequently, we show that the fee charged to ESG investors in the case there are only ESG funds in the market are always higher or equal than those in the case when both ESG and conventional funds coexist. Notice also that the equality holds if and only if $\lambda_N = 0$, i.e. there are no neutral investors.

In the case of the neutral investors we have shown that $f_{HS}^O \geq f_{HS}$ and that $f_{HS} > f_{HC}$. As a result, the fee charged to the neutral investors when there are only ESG funds are higher or equal than those in the case when both ESG and conventional funds coexist. □

Lemma 4. *In the case when there are in the market both ESG and conventional funds, we* *have that the total welfare of ESG investors equals to*

$$W_{ESG} = \int_0^x U_{Pi} di + \int_y^{\lambda_P} U_{Pi} di = (R_H - f_S + u_0) \frac{x}{\lambda_P} - k \int_0^x \frac{x_i}{\lambda_P} dx_i + \tag{8}$$

$$(R_L - f_{LS} + u_0) \frac{(\lambda_P - y)}{\lambda_P} - k \int_y^{\lambda_P} (\lambda_P - x_i) \frac{1}{\lambda_P} dx_i$$

$$= \frac{1}{\lambda_P} ((R_H - f_{HS} + u_0) x + (R_L - f_{LS} + u_0)(\lambda_P - y)) - \frac{1}{2\lambda_P} k x^2 - \frac{k}{2\lambda_P} (\lambda_P - y)^2$$

$$= \begin{cases} W^1_{ESG} & if \quad k \leq k_1, \\ W^2_{ESG} & if \quad k_1 < k < k_2, \\ W^3_{ESG} & if \quad k \geq k_2, \end{cases}$$

where

$$W^1_{ESG} = R_L + u_0 + \frac{1}{2} k \lambda_P,$$

$$W^2_{ESG} = \frac{1}{12} \frac{\Delta^2 + 12 k \lambda_P u_0 + 6 k \lambda_P (R_L + R_H) - 9 k^2 \lambda_P^2}{\lambda_P k},$$

$$W^3_{ESG} = \frac{1}{8} \frac{(R_H + u_0)^2}{k \lambda_P} + \frac{1}{8} \frac{(R_L + u_0)^2}{k \lambda_P}.$$

In the case there are only ESG funds in the market the total welfare of ESG investors is equal to

$$W^O_{ESG} = \int_0^{x^O} U_{Pi} di + \int_{y^O}^{\lambda_P} U_{Pi} di = \left(R_H - f^O_{HS} + u_0\right) \frac{x}{\lambda_P} - k \int_0^{x^O} \frac{x_i}{\lambda_P} dx_i + \tag{9}$$

$$\left(R_L - f^O_{LS} + u_0\right) \frac{(\lambda_P - y^O)}{\lambda_P} - k \int_{y^O}^{\lambda_P} (\lambda_P - x_i) \frac{1}{\lambda_P} dx_i$$

$$= \frac{1}{\lambda_P} \left(\left(R_H - f^O_S + u_0\right) x^O + \left(R_L - f^O_{LS} + u_0\right)\left(\lambda_P - y^O\right)\right) - \frac{1}{2\lambda_P} k \left(x^O\right)^2 + \frac{k}{2\lambda_P} \left(\lambda_P - y^O\right)^2$$

$$= \begin{cases} W^{O1}_{ESG} & if \quad k \leq k^O_1, \\ W^{O2}_{ESG} & if \quad k^O_1 < k \leq \min\left\{k^O_2, \frac{\Delta}{2\lambda_N}\right\}, \\ W^{O3}_{ESG} & if \quad k^O_2 < k \leq \max\left\{k^O_2, \frac{\Delta}{\lambda_N}\right\}, \end{cases}$$

where

$$\begin{aligned} W^{O1}_{ESG} &= R_L + u_0 + \frac{1}{2} k\lambda_P, \\ W^{O2}_{ESG} &= \frac{1}{36} \frac{k^2\left(4\lambda_N^2 - 45\lambda_P^2 - 36\lambda_N\lambda_P\right) + 18k\lambda_P\left(2u_0 + R_L + R_H\right) - 4k\Delta\lambda_N + \Delta^2}{k\lambda_P}, \\ W^{O3}_{ESG} &= \frac{1}{8}\frac{\left(R_H + u_0\right)^2}{k\lambda_P} + \frac{1}{8}\frac{\left(R_L + u_0\right)^2}{k\lambda_P} - \frac{1}{4}\lambda_N \frac{u_0 + R_H}{\lambda_P}. \end{aligned}$$

The welfare of investors satisfies the following inequalities: $W^1_{ESG} \geq W^2_{ESG} \geq W^3_{ESG}$*, and* $W^1_{ESG} = W^{O1}_{ESG}$*,* $W^2_{ESG} \geq W^{O2}_{ESG}$*,* $W^3_{ESG} \geq W^{O3}_{ESG}$*.*

Proof. Define $F(k) = \frac{1}{12}\frac{\Delta^2 + 12k\lambda_P u_0 + 6k\lambda_P\left(R_L + R_H\right) - 9k^2\lambda_P^2}{\lambda_P k}$. We calculate

$$F'(k) = -\frac{1}{12}\frac{9k^2\lambda_P^2 + \Delta^2}{k^2\lambda_P} < 0.$$

This implies that $F(k)$ is increasing and

$$F\left(\frac{\Delta}{3\lambda_P}\right) \geq F(k) \geq F\left(\frac{1}{2\lambda_P}\left(2u_0 + R_H + R_L\right)\right) \text{ for all } k_1 \leq k \leq k_2.$$

Notice that $W^1_{ESG} = F\left(\frac{\Delta}{3\lambda_P}\right)$ and $W^2_{ESG} = F(k)$.

Define also

$$G(k) = \frac{1}{8}\frac{\left(u_0 + R_H\right)^2}{k\lambda_P} + \frac{1}{8}\frac{\left(u_0 + R_L\right)^2}{k\lambda_P}.$$

It can be shown that

$$G'(k) = -\frac{1}{8}\frac{2u_0^2 + R_L + R_H + 2u_0R_L + 2u_0R_H}{k^2\lambda_P} < 0.$$

so for all $k \geq k_2$, we have that $G(k_2) \geq G(k)$.

Finally since

$$\begin{aligned} F\left(\frac{1}{2\lambda_P}\left(2u_0 + R_H + R_L\right)\right) &= \frac{1}{24}\frac{4\Delta^2 + 12u_0\left(u_0 + R_L + R_H\right) + 3\left(R_L + R_H\right)^2}{2u_0 + R_L + R_H}, \\ G\left(\frac{1}{2\lambda_P}\left(2u_0 + R_H + R_L\right)\right) &= \frac{1}{4}\frac{2u_0^2 + R_L + R_H + 2u_0R_L + 2u_0R_H}{2u_0 + R_L + R_H}, \end{aligned}$$

we have that

$$F\left(\frac{1}{2\lambda_P}\left(2u_0+R_H+R_L\right)\right)-G\left(\frac{1}{2\lambda_P}\left(2u_0+R_H+R_L\right)\right)=\frac{1}{24}\frac{\Delta^2}{2u_0+R_L+R_H}>0.$$

Therefore it implies that
$W_{ESG}^2=F\left(k\right)\geq F\left(\frac{1}{2\lambda_P}\left(2u_0+R_H+R_L\right)\right)>G\left(\frac{1}{2\lambda_P}\left(2u_0+R_H+R_L\right)\right)\geq W_{ESG}^3.$

Next we show that $W_{ESG}^1=W_{ESG}^{O1}$, $W_{ESG}^2\geq W_{ESG}^{O2}$, $W_{ESG}^3\geq W_{ESG}^{O3}$.

When both markets are covered the fees and quantities are the same, so the welfare is the same $W_{ESG}^1=W_{ESG}^{O1}=R_L+u_0+\frac{1}{2}k\lambda_P$.

When the funds HS and LS compete for investors we have that

$$\begin{aligned}W_{ESG}^2-W_{ESG}^{O2}&=\frac{1}{12}\frac{\Delta^2+12k\lambda_P u_0+6k\lambda_P\left(R_L+R_H\right)-9k^2\lambda_P^2}{\lambda_P k}\\&-\frac{1}{36}\frac{k^2\left(4\lambda_N^2-45\lambda_P^2-36\lambda_N\lambda_P\right)+18k\lambda_P\left(2u_0+R_L+R_H\right)-4k\Delta\lambda_N+\Delta^2}{k\lambda_P}=\\&\frac{1}{18}\frac{-2k^2\lambda_N^2+9k^2\lambda_P^2+\Delta^2+2k\Delta\lambda_N+18k^2\lambda_N\lambda_P}{k\lambda_P}>0.\end{aligned}$$

Finally, when the funds HS and LS are local monopolies

$$\begin{aligned}W_{ESG}^3-W_{ESG}^{O3}&=\frac{1}{8}\frac{\left(R_H+u_0\right)^2}{k\lambda_P}+\frac{1}{8}\frac{\left(R_L+u_0\right)^2}{k\lambda_P}\\&-\left(\frac{1}{8}\frac{\left(R_H+u_0\right)^2}{k\lambda_P}+\frac{1}{8}\frac{\left(R_L+u_0\right)^2}{k\lambda_P}-\frac{1}{4}\lambda_N\frac{u_0+R_H}{\lambda_P}\right)=\frac{1}{4}\lambda_N\frac{R_H+u_0}{\lambda_P}>0\end{aligned}$$

if $\lambda_N>0$, and $W_{ESG}^3=W_{ESG}^{O3}$ when $\lambda_N=0$. □

Proof of Proposition 4. We compare first the welfare of neutral investors. Since in both cases they are investing with a high quality fund and their utility is R_H-f_φ, $\varphi\in\{HS,HC\}$ and the fee charged by HC, f_{HC} is lower than the fee charged by the high quality ESG fund HS, f_{HS}, we can conclude that the welfare of neutral investors when there are only ESG funds is lower then when there are both ESG and conventional funds. Notice that the welfare of the neutral investors in the case conventional funds coexist with ESG funds is equal to

$$W_N = \int_0^{\lambda_N} U_{Ni} di = \lambda_N \left(R_H - f_{HC}\right),$$

while in the case only ESG funds exists and there is an equilibrium the welfare of neutral investors equals

$$W_N^O = \int_0^{\lambda_N} U_{Ni}^O di = \lambda_N \left(R_H - f_{HS}\right).$$

Let us next study the total welfare of ESG investors. In the case when there are in the market both ESG and conventional funds, we have that the total welfare of ESG investors is given by (8), while in the case when there are in the market only ESG funds the welfare is given by (9).

Notice that in case $k \geq \frac{\Delta}{k\lambda_N}$, there is no equilibrium in case only ESG funds exist and therefore the total welfare of ESG investors equals 0. We will therefore study the case $k < \frac{\Delta}{k\lambda_N}$. The next cases will be restricted to the case $k < \frac{\Delta}{k\lambda_N}$ and therefore, we consider only the intersection of the following cases with the case $k < \frac{\Delta}{k\lambda_N}$.

Case A $k_2^O < k_1$

Case A.1 $k < k_1^O$

Both markets are covered and $W_{ESG}^1 = W_{ESG}^{O1}$ it implies that $W_{ESG} = W_{ESG}^O$.

Case A.2 $k_1^O \leq k < k_2^O$

Market in case the conventional fund exist is covered and the welfare is $W_{ESG} = W_{ESG}^1$.

In case only the ESG funds exists the funds HS and LS compete for investors and the welfare is W_{ESG}^{O2}.

Since $W_{ESG}^1 \geq W_{ESG}^2$ and $W_{ESG}^2 \geq W_{ESG}^{O2}$ it implies that $W_{ESG} \geq W_{ESG}^O$.

Case A.3 $k_2^O \leq k < k_1$

The ESG market in case the conventional funds exist is covered and the welfare is $W_{ESG} = W_{ESG}^1$. In the case only the ESG funds exist, the funds HS and LS are local monopolies and the welfare is W_{ESG}^{O3}. Since $W_{ESG}^1 \geq W_{ESG}^3$ and $W_{ESG}^3 \geq W_{ESG}^{O3}$ it implies that $W_{ESG} \geq W_{ESG}^O$.

Case A.4 $k_1 \leq k < k_2$

In the case in which the conventional funds exist and the funds HS and LS compete for ESG investors investors, the welfare is W^2_{ESG}. In the case only the ESG funds exist, the funds HS and LS are local monopolies and the welfare is W^{O3}_{ESG}. Since $W^2_{ESG} \geq W^3_{ESG}$ and $W^3_{ESG} \geq W^{O3}_{ESG}$ it implies that $W_{ESG} \geq W^O_{ESG}$.

Case A.5 $k_2 \leq k$

In case the conventional funds exist in the market, the funds HS and LS are local monopolies and the welfare is W^3_{ESG}. In the case only the ESG funds exist, the funds HS and LS are also local monopolies and the welfare is W^{O3}_{ESG}. Since $W^3_{ESG} \geq W^{O3}_{ESG}$ it implies that $W_{ESG} \geq W^O_{ESG}$.

Case B $k_1 \leq k^O_2$

Case B.1 $k < k^O_1$

Both markets are covered and $W^1_{ESG} = W^{O1}_{ESG}$ it implies that $W_{ESG} = W^O_{ESG}$

Case B.2 $k^O_1 \leq k < k_1$

The market in case the conventional funds exist is covered and the welfare is $W_{ESG} = W^1_{ESG}$. In case only the ESG funds exist, the funds HS and LS compete for investors and the welfare is W^{O2}_{ESG}. Since $W^1_{ESG} \geq W^2_{ESG}$ and $W^2_{ESG} \geq W^{O2}_{ESG}$ it implies that $W_{ESG} \geq W^O_{ESG}$.

Case B.3 $k_1 \leq k < k^O_2$

In case the conventional funds exist in the market, the funds HS and LS compete for investors and the welfare is W^2_{ESG}. In case only the ESG funds exist, the funds HS and LS compete for investors and the welfare is W^{O2}_{ESG}. Since $W^2_{ESG} \geq W^{O2}_{ESG}$ it implies that $W_{ESG} \geq W^O_{ESG}$.

Case B.4 $k^O_2 \leq k < k_2$

In the case the conventional funds exist in the market the funds HS and LS compete for investors and the welfare is W^2_{ESG}. In the case only the ESG funds exists the funds HS and HL are local monopolies and the welfare is W^{O3}_{ESG}. Since $W^2_{ESG} \geq W^3_{ESG}$ and $W^3_{ESG} \geq W^{O3}_{ESG}$ it implies that $W_{ESG} \geq W^O_{ESG}$.

Case B.5 $k_2 \leq k$

In the case the conventional funds exist, the funds HS and LS are local monopolies and the welfare is W^3_{ESG}. In case only the ESG funds exist the funds HS and LS are local monopolies and the welfare is W^{O3}_{ESG}. Since $W^3_{ESG} \geq W^{O3}_{ESG}$, it implies that $W_{ESG} \geq W^O_{ESG}$.

□